Relation of Logic to Meta-Physics

1912

sciences, ignore the metaphysical side of the question—how far our experiences, *i. e.*, our sensations and feelings actually resemble and agree with things as they are in themselves independent of our sensations.

It is to be noted also that this view of Logic is not adopted in all logical treatises. The subject-matter of Metaphysical Logic is more extensive than that of the ordinary Logic. Metaphysical Logic is not distinguishable from Metaphysics proper. Lotze's Logic may be cited as its example.

"It cannot be pretended that logical principles can be altogether irrelevent to the abstruse discussions of philosophers, nor is it possible but that the view we are led to take of the problem which Logic proposes, must have a tendency favourable to the adoption of some one opinion on these controverted subjects, rather than another. For Metaphysics, in endeavouring to solve its own peculiar problem, must employ means, the validity of which falls under the cognizance of Logic. The moment the science of Metaphysics begins to draw inferences from evidence, Logic becomes the sovereign judge whether its inferences are well-grounded, or what other inferences would be ... This constitutes no nearer or other relation ... d Metaphysics, than that which exists between ... ther science."—Mill.

... and Psychology :—(1) As Psychology is ... om ...

—whether of thought, of feeling, or of willing,—and their relation to each other. Logic does not do so; it is concerned with thought only.

(ii) Psychology is concerned with the processes of conception, judgment and reasoning as they actually occur in the mind and attempts to systematize them, while Logic studies them with a view to regulate them for arriving at Truth. Psychology investigates the nature of thinking correct or incorrect; Logic furnishes criteria by which false reasoning may be discriminated from true. It does not enquire how men *do* think, but lays down laws in accordance with which they *should* think. So Psychology is empiricial and actual; and Logic is regulative and ideal. The regulative and ideal character of Logic is also to be found in what is called Empirical Logic.

(iii) Psychology takes account of thought and all other mental processes; but all its investigations are to be conducted with logical principles. In this respect it is narrower or less general than Logic, which governs the methods of all sciences.

(iv) Though the provinces of Psychology and Logic are thus distinct, yet the latter can only be satisfactorily studied in connection with the former. To thoroughly understand Logic, it is necessary to know what is the nature of the thinking mind, what are its limitations, what is the character of the process of thought, and how it unites with the other mental elements to form those concepts and judgments which are the materials with which Logic deals.

§ 3. Logic, Ethics and Æsthetics :—Logic, like Ethics and Æsthetics, is a regulative science. As Ethics assigns the conditions of right conduct and Æsthetics deter-

mines the principles of criticism and good taste, so Logic assigns the conditions of correct thinking and determines the principles of different kinds of thought. Logic springs from the Psychology of cognition; Ethics from the Psychology of Feeling and Æsthetics from the Psychology of Willing.

§ 4. **Logic and the Special Sciences**:—*Logic is the most general of all sciences.* Its aid is required in all the sciences. One name which has been given to Logic, namely the science of sciences, very aptly describes the all-extensive power of logical principles. The cultivators of special branches of knowledge appear to have been fully aware of the allegiance they owe to Logic, for that is shown by the very names of those sciences. The very name of Logic, says Jevons, occurs as part of nearly all the names adopted for the sciences, which are often vulgarly called the "ologies," but are really the logics; the "O" being only a connecting vowel or part of the previous word. Thus Geology is Logic applied to explain the formation of the earth's crust; Biology is Logic applied to the phenomena of life; Psychology is Logic applied to the nature of mind; and the same is the case with Physiology, Zoology and all other special sciences.

Though thus related to all sciences, yet Logic has the closest relations with those sciences which treat of Being, of Mind and of Language, for it investigates *thoughts about things expressed in speech.* But as we have seen, Logic does not investigate the truth, trustworthiness or validity of its own data and principles, this task belongs to Metaphysics, the criticism of knowledge and beliefs. As to the concrete sciences, such as Astronomy, Chemistry, Zoology, Politics,—

Logic is implied in them all; for all the propositions of which they consist, involve causation, co-existence and class-likeness. Logic is therefore said to be *prior* to them and *above* them, meaning by *prior* not that it should be studied earlier, for that is not a good plan; meaning by *above* not in dignity, for distinctions of dignity amongst liberal studies are absurd. But it is a philosophical idiom to call the abstract *prior to* or *higher than* the concrete; and *Logic is more abstract than Astronomy or Politics or such other special sciences.*

III.

THOUGHT, KNOWLEDGE, TRUTH, SCIENCE.

§ 1. **What is meant by Thought?**—The word "thought" has been used in three different senses.—(i) In popular language, the word "thought" means any mental state or phenomenon whether of thinking, feeling or willing. (ii) In the language of Psychology, it sometimes includes the cognitive factors of consciousness in general, *i. e.*, the element of cognition which is combined in all consciousness. (iii) As used in Logic, "thought" means sometimes the process and sometimes the product of comparision. In the former sense, it stands for, conception, judgment and reasoning; in the latter sense, it is a concept, or a judgment, or a reasoning.

§ 2. **Laws of thought.**—There are some laws which we have to observe whenever we think. These are called the Laws of Thought. They are three in number.

(i) The data must remain unaltered throughout the whole process of thinking. This law is called the Law of Identity.

(ii) Two contradictory terms cannot both be true at the same time of one and the same individual thing. This law is called the Law of contradiction.

(iii) One of two contradictory terms must be true, and both cannot be false at the same time of one and the same individual thing. This is called the Law of Excluded Middle.

§ 3. **Thought and Knowledge :**—Knowledge arises from "thought" involving comparison.

§ 4. **What is knowledge ?**—To have a knowledge of things means to have ideas corresponding adequately to the things and their qualities and relations with each other and with other things, accompanied by belief in their correspondence.

"Knowledge is made up of affirmations respecting the order of the world. These affirmations are the subject of belief, of which the ultimate criterion is Action."—Bain.

Twice two is four; the sun rises and sets; heat causes water to boil:—are affirmations or knowledge, respecting the universe. We believe them, and show our belief by acting on them. When we desire water to boil, we apply heat; which is our belief of the affirmation.

§ 5. **Knowledge should be true.**—An affirmation is true, when on actual trial, it corresponds to the fact. This is the direct proof. Indirectly we may test the truth of affirmations by comparing one with another. Whenever there is contradiction, there must be falsehood.

§ 6. **Limits of knowledge :**—According to Bain our knowledge is limited by our natural sensibilities. The knowable is limited by our senses and power of understanding.

§ 7. **Leibnitz's Analysis of the Character of knowledge :**—Knowledge is either *obscure* or *clear*. The clear is again either *confused* or *distinct*; and the distinct either *adequate* or *inadequate*; is further either *symbolical* or *intuitive*; and if it be at the same time both adequate and intuitive, it is *perfect*.

A notion is *obscure* when it is not sufficient to enable us to

recognize the thing which it represents :—when, for example, I remember some flower or animal which, I have formerly seen, but this remembrance is not sufficient to enable me to recognize its image, or to discriminate it from others which resemble it.

Knowledge is *clear*, when it enables us to recognize the thing represented, and to discriminate it from others which resemble it.†

Knowledge which is clear is said to be *confused* when we are not able to enumerate marks sufficient to discriminate the thing from others, although it may, in reality have such marks and requisites into which its notion may be resolved. Thus we see that painters and other artists discern well enough what is well or ill done; but often are not able to give a reason for their judgment, and reply to those who enquire what it is that displeases them in the work, that there is something they, knew not what, wanting.

Knowledge which is clear is said to be *distinct* when we know all marks and tests which are sufficient to distinguish it from all other similar bodies *i.e.* when we have the knowledge of the different parts of it. †

A distinct knowledge of an indefinable notion is, however, possible when it is primitive or self-evident—that is, when it is ultimate. Such a notion can only be understood *per se*, and thus wants the requisite of a definition. But in composite

† The definitions of *clear* and *distinct* knowledge as given by Prof. Sully in his Psychology is different from the definitions given here. What we call here *distinct* knowledge is according to him *clear*; and what we call here *clear* knowledge is according to him *distinct*.

notions, in-as-much as the individual component marks are known—sometimes clearly indeed, but nevertheless confusedly—such as weight, colour and insolubility in aqua-fortis, and others which enter into the notion of gold,—such a knowledge as this of gold, though it be distinct, is nevertheless *inadequate*.

When everything which enters into a distinct notion is distinctly known, or when the last analysis is reached, the knowledge is *adequate*, of which a perfect example is not obtainable—the knowledge of numbers approaches near to it.

Sometimes we do not behold at a glance the whole nature of the thing, but employ *signs* instead of *things*. We commonly omit, for the sake of expedition, any explication of these signs in present thought, knowing or believing that we have such explication in our power. Such knowledge is called *symbolical*, which is the kind that we employ in Algebra and Arithmetic. Our knowledge of a polygon of a thousand equal sides is of this character. Our knowledge is *symbolical*, when a notion is very complex, when we are not able to think together at once all the notions which make it up.

When we are able wholly or at least to a great extent, to think together at once all the notions making up another notion, our knowledge is then called intuitive. Our knowledge of distinct primitive notions is always simply intuitive.

§ 8. **Knowledge, Belief and Opinion** :—If we consider truth by relation to the degree and kind of certainty, we have to distinguish *knowledge*, *belief* and *opinion*. Here the word "Belief" is used in its popular sense and not in its psychological sense.

Knowledge and *Belief* differ not only in degree, but in kind.

Knowledge is a certainty founded upon insight: *Belief* is a certainty founded upon feeling. The one is perspicuous and objective, the other is obscure and subjective. Each however supposes the other; and an assurance is said to be a knowledge or a belief according as the one element or the other preponderates. *Opinion* is the admission of something as true when neither insight nor feeling is so intense as to necessitate a perfect certainty.

§ 9. **Doubt and Probability**:—What prevents the admission of a proposition as certain, is called *Doubt*. The approximation of the imperfect certainty of opinion to the perfect certainty or belief is called *Probability*.

§ 10. **Knowledge, a posteriori and a priori**:—Knowledge may be divided into two classes considering its origin,—(i) Empirical or a posteriori, and (ii) Intuitive or a priori. This distinction is metaphysical.

Empirical knowledge is derived from experience *i.e.* from the presentation of sense, external and internal. **Intuitive knowledge** is not obtained from experience. The doctrine, therefore, of the acquisition of knowledge must consist of two parts, the first treating of the acquisition of knowledge through the data of experience; the second, of the acquisition of knowledge through the data of intelligence.

§ 11. **Knowledge, Particular or General**:—Knowledge is either Particular or concrete, or General or abstract.

The knowledge of a certain individual thing as a tree or a house or any other object actually present to our senses is particular and concrete. The knowledge respecting a whole class or species of things is general and abstract.

§ 12. General knowledge is useful.—It is desirable to attain knowledge in the highest degree of *generality*. The reason is obvious. A general affirmation is a great many particular affirmations in one. It is a vast economy of the human understanding. A general law enables us to survey by one glance, a wide array of facts.

§ 13. Bacon on the acquisition of knowledge:—Bacon regards the acquirement of knowledge as an interpreting of nature or law, which is a gradual process. He condemns the scholastic method of discovering truth. This, which he calls 'anticipating nature,' he thus describes: "It leaps from the senses and particulars to the most general axioms." In contrast with this his own method of "interpreting nature" is one which raises axioms from the senses and particulars by ascending steadily, step by step, so that at last the more general may be reached.

According to Bacon, "anticipating nature" is unphilosophical. This view is an attack on hypothesis. This is the weakest point in Bacon's system. Without hypothesis, science cannot make progress; discovery becomes impossible. Bacon wanted to level all intellects.

§ 14. Bacon's Idolas:—Bacon is not wrong in thinking that the mind must be cleared of the 'phantoms' or 'idolas' or false conceptions which are deeply rooted in man's intellect. Of these phantoms or idolas, Bacon enumerates four classes of which the first two are intrinsic and the second two extrinsic:—(1) *Idola Tribus* or *phantoms of the Tribe*. These are the mistakes arising from false interpretation of the impressions received through senses. (2) *Idola specus* or *phantoms of the*

Cave. These are the mistakes arising from individual peculiarities,—peculiar education, particular frame of mind. (3) *Idola Fori* or *phantoms of the market-place.* These are the mistakes arising from the intercourse and society of men with one another, *i.e.* errors due to language. We have words indicating things which do not really exist; at last we think them to be existing. (4) *Idola Thei* or *phantoms of the Theatre.* These are the mistakes arising from the false theories accepted. They are the illusions from the brain, the false systems of philosophy.

§ 15. **Empirical knowledge and Inductive Logic**:—To lay down the method of testing our Empirical knowledge is the main business of Inductive Logic. Inductive Logic is concerned with our Empirical knowledge. It has not much to do with Intuitive truths or truths known immediately. So Inductive Logic is also called Empirical Logic.

§ 16. **Truth: What it is.**—Truth is the correspondence or agreement of a cognition with its object. All admit that by "Truth" is understood a harmony, an agreement, a correspondence between our thought and that which we think about.

§ 17. **Error: What it is.**—Truth has been said to be the agreement of thought with its object. Error is the opposite of Truth; Error must necessarily consist in want of this agreement.

§ 18. **What is meant by a Science?**—Science means systematized knowledge. The perfect form of knowledge is Science. A Science is (*a*) a collection of general truths or a systematic attempt to arrive at such a collection, (*b*) con-

cerning some particular department of the universe as it appears to us, (*c*) stated in such a way as to show how they are connected with and depend upon one another.)

§ 19. The peculiarities of Science :—

I. It employs special means and appliances to render knowledge true. These are the logical methods of determining truth.

II. Knowledge, in the form of Science, is made as *general* as possible. Science does indeed endeavour to determine accurately individual facts, but this it does with a view to arrive at the most general truths regarding them. A few isolated facts carefully ascertained to be true, would be valuable in themselves, but they would not constitute a Science.

III. A Science embraces a *distinct department* of the world, or groups together facts and generalities that are of a kindred sort. The world of reality represents phenomena widely varying in character and consequently the affirmations or truths regarding them should be grouped separately.

IV. A Science has a certain *order* or arrangement of topics, suitable to its ends in gathering, in varying, and in communicating knowledge. Every Science has a particular method of procedure for observing, testing and communicating the truths ascertained. Still in all of them attention must be given to the following points :

(*a*) It is necessary to proceed from the more easily to the less easily known. If any fact or generality depends upon or presupposes another, that other should be stated first in order.

(*b*) Whatever is requisite for proving any doctrine should precede what is to be proved.

(*c*) The meaning of all terms should be distinctly given before they are made use of. It is useful to commence with the definitions of leading terms.

§ 20. **Sciences: Theoretical and Practical:**— In a theoretical science, we obtain, in the most succinct and intelligible shape, the entire body of existing informations relating to one group of kindred phenomena. In a practical science we learn how informations regarding phenomena may be utilized for practical purposes. A theoretical science teaches us *to know*; a practical science teaches us *to know how to do*. Logic is both theoretical and practical.

§ 21. **Sciences: Positive and Regulative:**—A positive science only investigates matters of fact. Its business is to describe and inform facts as they are. A regulative science sets forth an ideal and points out what forms facts ought to assume. Psychology is a positive science; Logic is a regulative science, in-as-much as it regards Truth as an end desired, and points out some of the means of attaining it.

§ 22. **Sciences: Abstract and concrete:**— Logic and Mathematics are called abstract sciences; while Chemistry, Zoology and such other sciences are concrete. The Abstract sciences are the sciences of the relations in which things stand to one another, whatever the particular things may be that are so related. Each of the concrete sciences is concerned with a particular class of things. The abstract sciences properly precede the corresponding concrete sciences.

§ 23. **Sciences: Deductive and Inductive:**— Deductive sciences are those in which the reasonings used are mainly Deductive; and Inductive sciences are based on

observation and experiment through the use of the senses and direct consciousness—and on inductive inferences from such facts of experience. Mathematics and Political Economy are Deductive sciences; Chemistry and Botany are Inductive sciences. There has been a tendency in recent times to restrict the term "science" to Inductive sciences only. This narrow signification of science would exclude Deductive sciences from the category. Logic is both Deductive and Inductive.

§ 24. **Formal Knowledge, Formal Truth and Formal Science**:—There are for human thought only two species of object. For that about which we think must either be a thought or something which a thought contains. On this is founded the distinction of Formal knowledge and Real knowledge,—of Formal Truth and Real Truth. The knowledge of the *form* of thought is a Formal knowledge; and the harmony of thought with the *form* of thought is a Formal Truth. Thus self-consistency or freedom from contradiction is the test of Formal Truth. Formal Truth is Subjective.

Formal Knowledge is of two kinds; for it regards (i) the conditions of thought proper and also (ii) the conditions of our presentations or representations of external things. So **Formal Truth** is of two kinds:—(i) Logical and (ii) Mathematical. We have two **Formal Sciences**:—(i) Logic or at least Pure Logic; and (ii) Mathematics.

§ 25. **Real Knowledge, Real Truth and Real Science**:—**Real Knowledge** is the knowledge of the reality of the relation existing between objects of thought. **Real Truth** is the harmony between a thought and its matter.

Agreement with objective facts is the test of Real Truth. Real Truth is Objective.

Real Truths are divided into three classes :—(i) Metaphysical Truth, which denotes the harmony of thought with the necessary facts of mind ; (ii) Psychological Truth, the harmony of thought with the contingent facts of mind ; and (iii) Physical Truth, the harmony of thought with the phenomena of external experience.

The **Real Sciences** are the sciences of fact, for the point from which they depart, is always a fact, always a presentation ; they are conversant about existences other than the forms of thought. Some of these rest on the presentation of self-consciousness or the facts of mind ; others on the presentation of scientific perception or the facts of nature. The former are the mental sciences, the latter the material.

§ 26. What kind of science is the Material Logic ?—All science, all knowledge is divided into two great branches ; for it is either (1) conversant about the objects known or (2) conversant about the laws or conditions under which such objects are cognisable. The former of these is the *direct science* or science simply ; the latter, *Reflex science,*—the science of science or the method of science.

All Real Sciences are conversant about objects known. The science of science falls into two branches. Of these, one which treats of those conditions of knowledge which lie in the nature of thought itself is called *Pure Logic* ; the other, which treats of those conditions of knowledge, which lie in the nature, not of thought itself, but of that which we think about. The science, which expounds the laws by which our scientific

procedure should be governed, in so far as these lie in the forms of thought is called *Formal* or *Subjective* or *Abstract* or *Pure Logic*. The science which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the contents, materials, or objects, about which knowledge is conversant, is called *Material*, or *Objective* or *Applied* Logic. So the material Logic has a unique position among all sciences. It is not a material science; but closely connected with all material sciences;—it is the science of all such sciences.

Mill desired to extend the limits of logical method so as to test the material truth of propositions; he thought that he could expound a method by which experience itself and the conclusions of the special sciences may be examined. According to Carveth Reid, Logic is a Formal Science, and Induction may be treated formally; but logical forms are only valuable so far as they represent the actual relations of natural phenomena.

IV

IMMEDIATE AND MEDIATE KNOWLEDGE. INTUITIVE AND INFERENTIAL TRUTHS.

§ 1. **Immediate knowledge**:—Immediate knowledge is knowledge obtained without any mediating process *i.e.* through direct observation of the things themselves, and through the direct revelation of reason.

Immediate knowledge may be sensuous or rational.

(1) Immediate knowledge is sensuous when it is given through the senses or by introspection.

Sensuous knowledge has the following characteristics:— (*a*) It is always particular and singular. (*b*) It belongs to the present moment in time.

(2) Immediate knowledge is rational when the knowledge is obtained through reason or the faculty of rational Intuition. Reason or the faculty of rational Intuition reveals either universal laws that cannot be established by experience, or metaphysical realities.

The Rationalists and Intuitive philosophers believe in the existence of a faculty or reason or Intuition; while the Empiricists deny its existence.

§ 2. **Mediate Knowledge**:—Knowledge arrived at by some form of Inference, deductive or inductive, whether performed implicitly and unconsciously, or explicitly and with clear consciousness is mediate knowledge.

Mediate knowledge may be mistaken for immediate knowledge when the process of inference involved are rapid and unconscious.

§ 3. Mediate and Immediate knowledge:—Knowledge is mediate when it arises from inference; when it arises without inference, it is called immediate knowledge.)

§ 4. What is meant by Intuition?—(1) The term "Intuition" is used in the widest sense as identical with *immediate knowledge,* sensuous and rational, singular and universal. Every piece of knowledge obtained by direct consciousness is called, in this sense, *intuitive.*

(2) Generally the word "intuition" is used in a narrower sense and means discerning metaphysical realities and some general truths, through the direct and immediate operation of our mind, without any process of reasoning at all.

§ 5. Intuitive and Inferential Truths:—Intuitive truths are those which are known immediately by direct consciousness; and those known by the mediation of other truths are called Inferential Truths;—they are inferences whether from particular facts to general truths (Inductive), or from general truths to less general or individual one (Deductive).

§ 6. Logic does not discuss the questions regarding Intuitive Truths:—Facts of present consciousness, as—I am hungry, I hear a sound, I am pleased, I am speaking,—are amenable to no laws or rules; they are final and conclusive of themselves. We cannot escape from them, we cannot be more or less convinced of them by any method of procedure. They are the ultimate data of each one's knowledge. Intuitive truths are self-evident. Whatever is known to

us intuitively, by direct consciousness is known beyond possibility of question. What one sees or feels, whether bodily or mentally, one cannot but be sure that one sees or feels. No science is required for the purpose of establishing such truths; no rules can render our knowledge of them more certain than it is in itself. There is no logic for this portion of our knowledge.

§ 7. **Logic and Inferential Truths**:—Logic deals with inferential truths only. Logic has to determine whether our inferences are valid or otherwise. Logic enters into all questions which are to be answered for this purpose.

§ 8. **Difficulty in distinguishing Intuitive and Inferential Truths**:—"There are certain things admitted by all to be matters of intuition or immediate consciousness, such as our sensations and emotions in their primitive character; and certain other things equally admitted to be matters of inference or mediate cognition, such as the feelings of other men, the facts of testimony and the generalizations of science;—there is a middle ground or margin, where intuition and inference are blended and confused, and where what is accounted intuition by one man, may be called inference by another. This happens with some of the celebrated questions. The existence of the Deity is reckoned by some to be an intuition or an immediate revelation of consciousness, a judgment *a priori*; by others an inference from design or a judgment *a posteriori*: while most commonly it is viewed as both the one and the other. Again our perception of a material world is accounted an intuition by Reid and Hamilton; while others deny it to be intuitive in the sense intended. In fact the

controverted questions relating to the origin of our knowledge all lie upon the doubtful margin of intuition and inference."—Bain.

V

OBSERVATION AND EXPERIMENT.

§ 1. Observation and Experiment: The groundwork of Science.—Science is based upon matters of fact ascertained by *observation* and *experiment*. These supply us with the data upon which the generalisations of science, which embody the laws or uniformities of phenomena, are founded.

§ 2. Why Observation and Experiment are necessary for logical purposes.—In order to form a concept, we have to observe things or facts. When we judge, we compare them. Comparison is impossible without observation; consequently good reasoning depends to a great extent upon good observation. When some facts or phenomena appear before us and we have to infer any truth from them, we must observe them properly. So good observation is necessary for correct reasoning. We cannot have any experience unless we observe nature; so inferences based on experience cannot be had without observation. Experiment is a kind of observation. Here *observation* is distinguished from *Experiment*. Pure *Observation* is merely noticing events and changes which are produced in the ordinary course of nature, without being able or at least attempting to control or vary those changes; in *experiment* on the contrary we vary at our will the combinations of things and circumstances, and then observe the result. For the purpose of varying the circums-

tances,—that is, of obtaining a number of different kinds of instances of the phenomenon in question, we must have recourse to either *observation* or *experiment*; that is, we must either find suitable instances in nature, or we must *make* them by an artificial arrangement.

Without observation, we cannot form Theories or hypotheses as to facts. Observation is also necessary in order to *establish* a theory.

§ 3. Observation: External and Internal.—The external world is observed by means of the senses; and the internal world of mind by means of introspection or reflection.

§ 4. Relation of Observation and Experiment.—All observation implies isolation—mental if not physical—of the phenomena examined. Attention is concentrated on some elements of the complex totality of presentation, and is withdrawn from all others. This isolation is perfect and permits of definite and exact observation, only when all the material conditions of the phenomenon under consideration are fully and exactly known. But this is very frequently not the case. Nature presents us with complex totalities in which the pure connexion we are seeking, is obscured by the many extraneous elements with which it is mixed up.

And further, natural processes are in some cases so extremely slow and gentle that they escape observation. The decomposition of water was continually going on, although it had never been observed before the time of Lavoisier. It is evident therefore that it is frequently necessary for the observer to himself determine the conditions under which

he will examine the phenomena. Jevons says :—"To observe with accuracy and convenience we must have agents under our control, so as to raise or lower their intensity; to stop or set them in action at will." Such a definitely determined *observation* is called an *experiment*. The end aimed at in observation is full and exact knowledge of all operative conditions and this can frequently be attained only in experiment.

It is evident then that the difference between *observation* and *experiment* is one, not of kind, but of degree. Experiment is not a distinct method of acquiring knowledge, but is rather the preparation of the phenomena under consideration, so that the observation may be made under known, instead of under unknown conditions, and shall consequently attain the highest degree of accuracy. When complete knowledge of the essential conditions can be obtained by observation, experiment is unnecessary. In fact the obvious distinction between *observation* and *experiment* is that the latter is an immense extension of the former.

§ 5. When the observation is made with the help of a scientific instrument, is it simple observation or experiment?—The use of a scientific instrument does not transform an observation into an experiment, unless the instrument modifies the object which is being observed. Thus we invariably speak of *observing* with a telescope or a microscope. The fact is then that *experiment* is not merely *observation* under artificial and determinate conditions, but under artificial and determinate conditions which constitute an integral part of the image or product to be ob-

served.] Thus common dissection is not *experiment* though it introduces conditions in the way of separation and demarcation as definite as anything can be; but vivisection is experiment, because the determinate conditions it produces, enter as factors into the action of the organism observed.

§ 6. What is "Natural Experiment?"—The transition between observation and experiment is a gradual one, as might be expected from the fact that they differ only in degree. *Natural Experiments* are intermediate between Simple Observation and Pure Experiment. When the earliest astronomers simply noticed the ordinary motions of the sun, moon and planets upon the face of the starry heavens, they were pure observers. But astronomers now select precise places for important observations of the transits of planets. Natural experiments are regarded more as *Observations* than as *Experiments*.

§ 7. The Advantages of Experiment.—(1) Experiment enables us to obtain innumerable combinations of circumstances which are not to be found in nature and so add to nature's experiments a multitude of experiments of our own. Experiment is rendered indispensable by the fact that on the surface of the earth we usually meet substances under certain uniform conditions, so that we could never learn by observation what would be the nature of such substances under other conditions.

(2) In simple Observation one has to wait till suitable instances can be found and this involves much delay even when such instances are ultimately forthcoming, which they very often are not.

(3) When we can produce phenomenon by experimentation, we can take it, as it wer home with us and observe it in the midst of circumstances with which in all other respects we are accurately familiar.

(4) Experiment is absolutely necessary, when simple observation alone will not mak plain all the essential conditions of a phenomenon.

(5) By Experiment, knowledge can be advanced much more rapidly and surely than would be possible, if we had been confined to simple Obs rvation; for in the latter case the chief difficulty is to find ou what conditions are operative, and this often can not be done with precision, whilst it is the very point which the experimenter determines for himself.

(6) Experiment is our resource when we wish to determine the effect of a given cause, for we can take a cause and try what it will produce.

§ 8. The Advantages of Observation.—(1) Observation is chiefly applicable when we are unable to obtain artificial experiments. Some natural processes are so slow that Experiment is impossible.

(2) Observation often acquaints us with broad characteristics of phenomena, which in Experiment would have been obscured by special conditions.

(3) Observation is useful in investigating the unknown causes of a given effect.

§ 9. What are the different ways by which we can "improve our powers of observation"?—In order to consider this question we have to start from the dual standpoint; *viz.* from that which postulates an observer

and an object, the former endowed with various faculties of sense, the latter possessing various attributes. This seems to suggest three different ways in which we might conceivably improve our powers of observation; for (1) we might simply endeavour to get nearer to the object in question, so that the sense might be put to less strain in cognizing it; (2) we might endeavour to enlarge the object, or in some other way to intensify its operation upon the senses; or (3) we might confine the improvement to our sensitive powers by endeavouring to make these more delicate and refined.

§ 10. What is the relation of Hypothesis to Observation and Experiment?

Observations and Experiments are regulated by Hypothesis. The facts to be observed and experimented on are infinite; we must have a distinct purpose for our observation and experiment. A hypothesis always directs our observation to particular facts and lead us to experiments on them.

Hypotheses are formed also after some amount of observation.

§ 11. Fallacies of Observation:—The process of ascertaining what consequents, in nature, are universally connected with what antecedents, or in other words what phenomena are related to each other as causes and effects, is in some sort a process of Analysis. This operation, which we call analytical, in-as-much as it is the resolution of a complex whole into the component elements, is more than a mere mental analysis. The mental analysis however must take place first. And every one knows that in the mode of performing it, one intellect differs immensely from another. It

is the essence of the act of observing, for the observer is not he who merely sees the thing which is before his eyes, but he who sees what parts that thing is composed of. To do this well is a rare talent.

(i) One person *from inattention* or *attending only in the wrong place*, overlooks half of what he sees.

(ii) Another sets down much more than he sees, *confounding it with what he imagines* or *with what he infers*.

(iii) Another takes note of the *kind* of all the circumstances but being inexpert in estimating their degree, *leaves the quantity of each vague and uncertain*.

(iv) Another sees indeed the whole, but *makes such an awkward* division of it into parts, throwing things into one mass which require to be separated, and separating others which might more conveniently be considered as one.

Consequently fallacies of Observation arise

Fallacies of Observation are of two kinds

(1) Non-Observation and (2) Mal-Observation.

(1) Non-Observation :- It is Non-Observation, when all the error consists in overlooking or neglecting facts or particulars which ought to have been observed.

All observation implies selection and isolation of phenomena. In making this selection it is evidently easy to overlook or to neglect (i) either instances or (ii) conditions which are pertinent to the matter in hand.

Neglect of instances is frequently due to bias. We have a natural tendency to dwell upon instances which make for the theory we would like to see established, or which we have hitherto held, and to treat as of no importance those

which make against it or even to neglect to consider them at all. A very common form in which non-observation of instances appears is when attention is directed to positive instances whilst negative instances are neglected. This originates in a kind of natural tendency of the mind to be impressed by any occurrence and to omit to notice non-occurrence.

Neglect of operative conditions happen because in every examination of concrete reality a residue of unanalysed elements remains and this opens a possibility of leaving out of consideration some element which is an essential part of the phenomena under investigation. This fallacy is specially common in inferences on social and economic subjects.

(2) Mal-Observation :—It is Mal-Observation when something is not simply unseen, but seen wrong; when the fact or phenomenon, instead of being recognized for what it is in reality, is mistaken for something else. By Mal-Observation is meant the wrong interpretation of our impressions.

All observation involves implicit inference or the interpretation of what is received through the senses by referring it to some part of the knowledge previously possessed. When this reference is wrongly performed, we have a case of Mal-Observation. The rustic who takes a tombstone brightened by the rays of the moon for a ghost, falls into the fallacy we are now considering. The sensuous impression as received is just what it should be under the circumstances; in the meaning given to that impression the error will always be found to lurk. All illusions are cases of Mal-Observation.

VI.

ANALYSIS AND SYNTHESIS.

1. Analysis and Synthesis defined :— Analysis is the process of separating a whole into its parts, and Synthesis the combination of parts into a whole. Logical Analysis and Synthesis must not be confused with the *physical actions*, but they are nevertheless *actions of mind* of an analogous character. In the material world, both the separation and the combination are actual; in the mental world, both of them are ideal.

2. How do you distinguish Logical analysis and Synthesis from Chemical analysis and Synthesis ?

Chemical analysis as distinguishable from Logical analysis is the actual breaking up of a compound substance (*e. g.* water) into its elementary ingredients (*i. e.* oxygen and hydrogen). Chemical Synthesis is a reconstruction of compound material objects from the elementary substances into which they have been analysed. The recombination of oxygen and hydrogen to form water is an example of Chemical Synthesis. The Chemist who receives a drug for examination and decomposes it into several Chemical elements and ascertains their nature and comparative quantities, does what is called Chemical Analysis. When he brings together carefully weighed quantities of certain simple substances and makes them Chemically combine, he does what is called Chemical Synthesis. But Logical

Analysis is ideal separation, and Logical Synthesis is ideal combination *i. e.* they are separation and combination of parts of a thing or phenomenon, in thought only.

§ 3 Why Analysis and Synthesis are necessary for logical considerations?

(When we observe anything or phenomenon, we separate it out from the whole universe; our attention is directed to one thing only, so there is analysis in every case of observation. Then again when a thing is perceived, it is perceived as made up of several parts and having different phases so analysis and synthesis are both involved there. Without Analysis and Synthesis, Observation and Experiment cannot be effected. The processes of Analysis and Synthesis pervade every elementary operation which enters into Logic. We could not formulate propositions, we could not even conceive notions and employ terms, unless our minds and the minds of others before us had been long and actively at work in this way)

§ 4, Knowledge dependent upon Analysis and Synthesis: (Our knowledge of a thing is clear, when we can recognize it with certainty and do not confuse it with other things. (Our knowledge is distinct, when we can distinguish the parts and qualities of the thing known. In order to know the different parts of a thing *i. e.* to have a distinct knowledge of a thing, we must analyse it.)

Abstract ideas are formed by abstraction. Abstraction means taking asunder; so it is Synonymous with Analysis. (Abstraction consists in separating in thought the different qualities or parts of things from each other, which cannot be actually separated, and in trying to think of one or more of them by

Knowledge of a thing depends upon Synthesis also as impressions that we receive from a thing are synthesized. Dr. Ward writes :—" It is essential that impressions should recur and recur as they previously occured, if knowledge is ever to begin ; out of a continual chaos of sensation, all matter and no form, such as some philosophers describe, nothing but chaos could result. But a flux of impressions having this real or sense-given order will not suffice ; there must be also attention to and retention of the order. Out of the variety of impressions simultaneously presented, we do not instantly group together all the sounds and all the colours, all the touches and all the smells, but dividing what is given together, single out a certain sound or smell as belonging with a certain colour and feel, similarly singled out from the rest, to what we call one thing."

§ 5. Analytical and Synthetical Judgments :— The process of judging taken by itself seems to be a process of Synthesis only, as it consists in joining a new predicate idea to a subject idea, and combining them into one more compound idea. But we must remember that Judgments are generally divided into two classes :—(i) Analytic and (ii) Synthetic. When the connotation of the predicate of a proposition is the same as or a part of the connotation of the subject, the proposition is called Verbal or Analytical. When the connotation of the predicate is not a part of that subject, the proposition is called Real or Synthetical,

§ 6. Analytical and Synthetical Reasoning* :— Induction and Deduction are regarded respectively as analyti-

* This Section may be omitted on the first reading and may be read with advantage after reading the Notes on "Induction."

cal and synthetical reasoning. Analysis and Synthesis, as now understood, are almost universally regarded as the processes,—mental processes for the most part, but also extending to physical processes in a few cases, where it so happens that thinking can readily be translated, step by step, into action,—by which we break up a complex whole into parts, and put together a number of parts in order to constitute a whole. What do these words exactly denote in the province of Logic? To illustrate Synthesis, we might take the two simple premises, "All A is B," "All A is C," and combine them into the result "All A is BC." And to illustrate Analysis all we need do is to start with this conclusion and resolve it into the above two premises. Had premises and conclusions of this type been familiar to cultivators of common Logic, as they are to those of the Symbolic System, no one can hardly suppose that Synthesis and Analysis would ever have been applied to mark any other pair of operations than this.

The frequent employment of Analysis to indicate Induction dates, for English readers, from some well-known remarks by Newton in his "Optics", which have been quoted by D. Stewart and made the text of an interesting discussion. "As in Mathematics, so in Natural Philosophy, the investigation of difficult things by the method of Analysis ought ever to precede the method of Composition. This Analysis consists in making experiments and observations, and in drawing conclusions from them by Induction. By this way of Analysis we may proceed from compounds to ingredients; from motions to the forces producing them and in general from effects to their causes. This is the method of Analysis. And the

method of Synthesis consists in assuming the causes discovered and established as principles and by them explaining the phenomena proceeding from them and proving the explanations."

It is obvious that what Newton is here speaking of is something far wider than the mere logical processes indicated above. It is quite true that there is a large employment of Analysis in Induction. But so there is Synthesis too and they are both employed in Deductive reasoning. The fact seems to us to be that Analysis and Synthesis must be understood in a far wider sense than that in which we are able to identify them respectively with Induction and Deduction. *They are general processes, not confined to either of these forms of reasoning.* The utmost that we can say is that, *of the two, Induction decidedly makes much more use of Analysis than Deduction does.*

Induction may be regarded as analytical from one point of view; but may be regarded as Synthetical from another point of view. It is analytical, because from special cases we proceed to find out the fundamental. We go through particular cases to find out what is common to all. Induction is Synthetical, because by it we bind together all particular cases, not only observed but also unobserved. (Induction is analytic in *Intension, i.e.*, if we look to it from the attributive point of view; and Synthetic in *Extension, i.e.*, if we look to it from the denotative point of view.) Dr. Whewell says,—Induction is both analytical and Synthetical. In order to form a hypothesis, Analysis is required, and after that Synthesis.

Deduction is analytic in *extensive* point of view; because in deduction we pass from general to particulars; particulars appear to us. It is Synthetic in *intension.*

§ 7. Analytical and Synthetical. method:—(Correctness of inference is not all that is necessary either for the attainment of truth or for its exposition. It is also essential that those inferences should be properly combined. This is the subject-matter of Method.) The doctrine of Method considers how reasonings, when employed continuously upon any matter whatever, should be set forth to produce their combined effect on the mind. Method may therefore be defined as the correct arrangement of thoughts either for the discovery or for the exposition of truth.

The character of the method to be adopted is partly determined by the subject-matter dealt with, and partly by the object in view. This will be manifest from the discussion of Induction and its relation to Deduction. For in some cases we can begin with general judgments and principles and our object is to follow these out into their application in particular instances; and in other cases we must approach the problem from the side of particular instances and our object is to discover what general laws these particulars exemplify. There is thus a broad division of method corresponding to the distinction between inductive and deductive inference. When the object is to ascertain general principles, and the only procedure open to us is the examination of particular instances, the method is called Analysis. For here we start with the complex of experience, and by examination and thought try to analyse or resolve it into the simple and universal relations which are realized in it. On the other hand, when we start from general principles, either intuitively grasped or rigorously demonstrated, we follow the method of

Synthesis, for by the composition of simple and universal relations we seek to construct the complex individual.

So there are two methods :—(i) analytical and (ii) synthetical. These two methods are not opposed to each other. "They differ only as the road by which we ascend from a valley to a mountain does from that by which we descend from a mountain into a valley, which is no difference of road, but only a difference in going." A science is *experimental*, when its method is mainly analytic ; and *deductive* when its method is chiefly synthetic.

VII

CLASSIFICATION AND DEFINITION.

PART I.—*Classification.*

§ 1. **What is a Class?**—A class is a collection of individual objects having some attribute or attributes in common, which common attribute or attributes are connoted by the class-name. Every general name is the name of a class, for it is evident that when we invent a name connoting certain attributes, we thereby create a class consisting of everything possessing those attributes.

§ 2. **What is Classification?**—Classification, in its widest sense, is a mental grouping of facts or phenomena according to their resemblances and differences, so as best to serve some purpose. It is a mental grouping; for though in museums we often see the things themselves arranged in classes, yet such an arrangement only contains specimens representing a classification. The classification itself may extend to innumerable objects most of which have never been seen at all.

§ 3. **What is the principle of Classification?** Things are classed according to their resemblances and differences: that is to say, those that most closely resemble one another are classed together on that ground; and those that differ from one another in important ways are distributed into different classes. The more the things differ, the wider apart

are their classes both in thought and in actual arrangements. If their differences are very great, as with animals, vegetables and minerals, the classing of them falls to different departments of thought or science. We must not suppose that there is only one way of classifying things. The same objects may be classed in various ways according to the purpose in view. The principle of resemblance and difference is recognised in all these cases; but what resemblances or differences are important depends upon the purpose to be served.

§ 4. **Purposes of Classification**:—Purposes may themselves be classified: and here the most important distinction for the logical purpose is into *(a)* special or practical purposes, as in gardening or hunting, and *(b)* general or scientific, as in Botany or Zoology. The scientific purpose is merely knowledge, which may subserve all particular or practical ends, but has no other end than knowledge directly in view.

§ 5. **Uses of Classification**:—The general object of all classification is to keep our control over the facts by marshalling the objects in order; to know where to find a thing when it is wanted and to economise our statements in the retention and communication of our knowledge.

One use of classification is the better understanding of the facts of Nature or of any sphere of practice; for understanding consists in perceiving and comprehending the likeness and difference of things, in assimilating and distinguishing them; and in carrying out the process systematically new correlations of properties are continually disclosed.

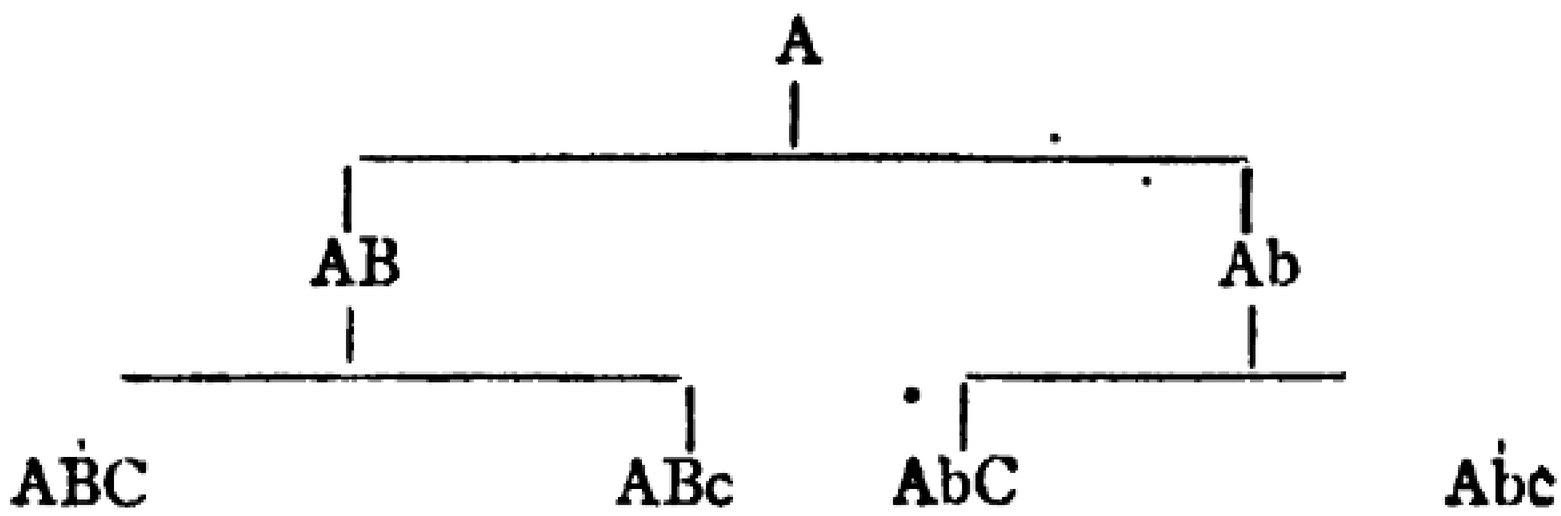
Another use of classification is to aid the memory. It

strengthens memory, because one of the conditions of our remembering things is that they resemble what we last thought of; so that to be accustomed to study and think of things in classes must greatly facilitate remembrance. But besides this, (it improves the character of memory, by making us more likely to remember what we want.) For what we want in any emergency is to remember what served the purpose in similar cases; or to recall cases similar to the present one.

§ 6. **Deductive and Inductive Classification**:—Classification may be either Deductive or Inductive; that is to say, in the formation of classes, as in the proof of propositions, we may, on the whole, (proceed from the more to the less, or from the less to the more general;) not that these two processes are entirely independent.

Deductive Classification is what is ordinarily called **Formal Division**. (The problem of division is concerned with the formal side and the problem of classification proper is concerned with the material side of the same process.) (Division and classification are the same thing looked at from different points of view) any table presenting a division presents also a classification. (A division starts with unity and differentiates it; a classification starts with multiplicity and reduces it to unity or at least to system.

The process of Deductive Classification or formal Division may be represented thus :



Given any class (A) to be divided.

1. Select one important character, attribute or quality (B), not common to all the individuals comprehended in the class, as the basis of division (*fundamentum divisionis*).

2. Proceed by *Dichotomy*; that is cut the given class into two, one having the selected attribute (B), the other not having it (b). This like all formal processes, assumes the principles of contradiction and excluded Middle.

When a class is thus divided, it may be called in relation to its sub-class, a *genus*; and in relation to it, the sub-class may be called *species* : thus,—Genus A ; species AB and Ab.

3. Proceed gradually in the order of the importance of characters ; that is having divided the given class, sub-divide on the same principle the two classes thence arising, and so again, *step by step*, until all the characters are exhausted. *Divisio ne fiat per saltum*.

The process of Inductive Classification may be represented thus :

Given any multitude of individuals to be classified :

1. Place together in groups (or in thought) those things

that have in common the most, the most widely diffused and the most important qualities.

2. Connect those groups which have, as groups, the greater resemblance and separate those that have the greater difference.

3. Demarcate, as forming higher and more general classes those groups that have important characters in common; and if possible, on the same principle, form those higher classes into classes higher still; that is to say, graduate the classification upwards.

Whilst in Division, the term *genus* and *species* are entirely relative to one another and have no fixed positions in a gradation of classes, it has been usual, in Inductive classification, to confine the term *species* to classes regarded as lowest in the scale and to give the term *genera* to classes on the step above, and at each higher step to find some new term, such as *Tribe*, *Order*, *Sub-kingdom*, *Kingdom*. If having fixed our species, we find them sub-divisible, it is usual to call the sub-species "*Varieties*."

§ 7. Connexion of Naming and Classification :—(*a*) General names have a meaning quite irrespective of classes. In predicating the name to a thing, we predicate only the attributes; and the fact of its belonging to a class does not in many cases come into view at all, when we assert a general name as *man* of a subject, we mean to assert that that subject possesses certain attributes "animality," "rationality" &c, which assertion has nothing to do with classes.

(*b*) But as soon as we employ a name to connote atributes

the things, be they more or fewer, which happen to possess those properties are constituted *ipso facto* a class. Every name which connotes an attribute divides, by that very fact, all things whatever into two classes, those which have the attributes and those which have not.

(*c*) Some times on the other hand general names owe their existence to classes. A name is sometimes introduced because we have found it convenient to create a class. This is usually the case with the classes of Plants and Animals; as when we divide Plants into Phænogamous and Cryptogamous.

§ 8. Relation of Classification to Inference:—All logical inference involves classification. Indeed every judgment involves classification, because the predicate is already a general term denoting what is common to a class. How then are classification and inference related? **Classification involves Inductive inference.** When we have a set of things A_1, A_2, A_3, &c before us and form them in our minds into a complex class by saying that all these A_s agree in having the property x we are forming a classification, and at the same time drawing an inductive generalisation. **Classification also involves Deduction.** When we refer a particular thing A_n to a certain class, or in other words, when we extend our class to include a new individual A_n, then we implicitly make a deductive inference.

As for example:—

A_1, A_2 A_3 &c.—Are members of a class, possessing the common attribute x.

A_n possesses the attribute x.

$\therefore$ A_n is also a member of that class.

§ 9. **The doctrine of Real kinds**:—Mill, following the schoolmen, says that there are in nature, Real kinds or such classes as are marked off from one another by an unknown multitude of distinguishing characteristics. Not-Real kinds are not so marked off. [The members of one real kind are characterised by the possession of an inexhaustible number of common properties, not referrible to any common cause.] The members of a Not-Real kind agree only in certain particulars which can be numbered,—that is, which have only certain specific and determinate common properties.

There are some classes, the things contained in which differ from other things only in certain particulars which may be numbered, while others differ in more than can be numbered, more even than we need ever expect to know. The former classes are not Real kinds; the latter are so.

Compare the class 'animals' with the class 'white things'; in the latter the members are not distinguished necessarily by any common properties except *whiteness*, and any properties,—effects of whiteness,—white things possess only a limited number of common properties and differ from other things only in respect of them; but a hundred generations have not exhausted the properties which are common to all animals; and though physiologists are continually discovering new ones, yet there is no probability that they will ever be able to say that they know of them all. Moreover these common properties are not referrible to any one cause. Animals differ from other things by many distinguishing features. The class 'animal' is a Real kind; the class of 'white things' is not. All animals possess an innumerable number of common properties; so

do all plants. The class of animals is marked off from the class of plants by an unknown multitude of distinguishing characters. So these two classes are two Real kinds.

Criticism:—Mill's doctrine of Real kinds is essentially modified by the modern theory of Evolution which does not recognise any absolute gap between things however different they may be in their nature, as Mill's doctrine seems to imply.

§ 10. **Real kinds and Natural Classes**:—It is sometimes said that Real kinds are *natural classes* or classes formed by nature; we have already seen that classes are really framed by the human mind, but according to Mill, the expression is true thus far,—(1) that Real kinds are classes for the recognition of which, as such, no elaborate process is generally required, because each of them is marked off from all others, not by some one or few properties which may be defficult to detect, but by its properties generally; and further (2) the end of classification would be subverted if we did not recognise Real kinds as classes.

§ 11. **Natural and Artificial Classification**:—Natural classification is based on those properties of things which indicate not only the most numerous but also the most important peculiarities. The properties, therefore, according to which objects are classified should, if possible, be those which are causes of many other properties; or at any rate which are sure marks of them—the properties which contribute most, either by themselves or by their effects, to render the things like one another, and unlike other things, which give to the class composed of them the most marked individuality and which could most impress the attention of a spectator who

know all their properties but was not specially interested in any.

By Artificial classification is meant any classification based on a more or less arbitrarily selected outward mark and peculiarity which do not necessarily correspond to or represent anything fundamental and essential in the nature of things. An alphabetical list of things is an example of such a classification.

"Natural classifications give us the deepest resemblances and relations and may lead us ultimately to a knowledge of the way in which the varieties of things are produced. They are therefore essential to a true science and may almost be said to constitute the frame-work of the science. Yet it does not follow that they are appropriate for all purposes. An artificial classification of the elements is thus necessary to the detection of substances and accordingly in any book on chemical analysis will be found arrangements of the elements according to characters of very minor importance, but which are selected on account of the ease and certainty with which they can be observed".—Jevons.

§ 12. The distinction of Classification as natural and Artificial criticised:—This distinction according to Dr. Venn is not a satisfactory one. (i) Every arrangement is artificial in so far as it is our own voluntary procedure and not a result offered to us from without. (ii) It is also artificial in the sense that we are seldom or never proposing to group the actual objects themselves, but only to make an ideal arrangement of them in our own minds. (iii) On the other hand every classification ought to aim at being natural in the sense of

conforming to the facts and endeavouring to be as suitable as possible to the circumstances.

§ 13. **Classification intended for general purposes and for special purposes:**—Dr. Venn does not like to make any such distinction as Natural or Artificial in classification; but the distinction which he likes to make is that between classification intended for special purposes and that which is intended for general purposes.

There must be some purpose or aim presupposed in every arrangement of the kind in question and the determination of this purpose at once puts a stamp upon the consequent classification. It is perfectly optional on our part to select our purpose, and the purpose may be of the most various kinds; but as soon as we have decided upon this, one particular arrangement is necessitated as being more complete and convenient, and therefore more *natural* than any other.

What is ordinarily regarded as an artificial classification, ought to be regarded, according to Dr. Venn, as a *classification for special purposes*; and what is generally known as a natural classification, ought to be regarded according to him, as *a classification for general purposes*. A special classification, being conditioned by this or that object or the want of this or that class of persons, is in a measure a personal or subjective one, so a truly objective classification, which shall hold good for all persons, shall deserve to be called a natural one.

§ 14. **What does Mill mean by Natural Groups? what according to him is the object of a Classification into such groups;**—Natural groups are

classes of such a kind that the things included therein resemble each other most in the general aggregate of their properties.

The object of a classification into such groups is the best possible ordering of our ideas in reference to the things, or to make us think of those objects together which have the greatest number of important common properties.

Its general problem is to provide that the things be thought of in such groups and those groups in such an order, as will best conduce to the ascertainment and remembrance of their laws.

§ 15. How are Natural groups constituted,—by type or by definition?

Whewell's theory is that Natural groups are constituted by *type* and not by *definition.*

We must understand what is meant by a "type." A type is an eminent example of any class *i.e.* an example which presents the characteristics of the class most conspicuously and completely. Natural classes, according to Whewell, are formed by being gathered round these types; and a class really consists of the type and all objects which bear a certain amount of general resemblance to the type.

The meaning of Whewell's theory is that objects are aggregated into Natural groups on the basis of *more general resemblance,* that is what Whewell calls by reference to a type and not by resemblance in specific assignable particulars, which can be expressed in a definition.

Mill's. Criticism of Whewell's Theory:—The truth is that every genus or family is framed with distinct reference to certain characters (*i.e.* by Definition which enumerates those

characters), and is composed, first and principally, of species which agree in possessing all those characters. Our conception of the class continues to be grounded on the characters; and the class might be defined;—those things which either possess that set of characters or resemble the things that do so, more than they resemble anything else. Not only, therefore, are Natural groups, no less than any artificial classes, determined by characters; they are constituted in contemplation of and by reason of characters.

But there is truth in Whewell's theory:—(i) It is not necessary for every member of a natural group to possess *all* the characters laid down as those of the group; and so far the definition may be said to fail in determining the group. In fact, natural classes might be defined in this way—those things which either possess such and such characters—(those enumerated in the definition), or resemble those things which do possess them more nearly than they resemble anything else.

(ii) Our general conception of the group is a type, to which we usually in the first instance refer as a ready means of suggesting to what group any given individual or species will most probably belong; but a *determination* of the question must always rest upon a reference to the characters laid down in the definition of the group. Natural grouping may, then, be said to be *suggested* by type (*i.e.* by mere resemblance), but *determined* by definition (*i.e.* by possessing specific characters or properties).

§ 16. **What is Classification by Series?**—Classification to be complete does not stop with arranging things

into groups; it also endeavours to arrange these groups in a serial order. Classification by series brings into one class all the things which exhibit the phenomenon which forms the basis of our classification in the same degree, and into another class all the things which exhibit it in a slightly higher degree, and so on till we have arranged a series of classes which present every manifestation of the phenomenon from the lowest to the highest and most complete.

§ 17. The Requesites of a Classification by Series:—(1) It requires the power of recognising the essential similarity of a phenomenon in its minuter degrees and obscurer forms with phenomneon in the greatest perfection of its development; that is, of identifying with each other all phenomena which differ only in degree, and in properties which we suppose to be caused by difference of degree.

(2) In order to recognise this identity or in other words, their exact similarity of quality, the assumption of a type-series is indispensable.

(3) It is necessary to bring into one grand class all kinds of things which exhibit the phenomenon, in whatever variety of form or degree.

(4) It is necessary to arrange these kinds into a series, according to the degree in which they exhibit it; beginning with those which exhibit in the greatest intensity and perfection, and terminating with those which exhibit least of it.

PART II.—*Definition.*

§ 1. Definition and Classification: How they are related.—The process of determining a definition is inseparable from classification. We know that classification consists in distributing things into groups according to their likenesses and differences, regarding as a class those individuals which have most qualities in common. In doing so we must of course, recognise the common qualities or points of likeness; and to enumerate these is to define the name of a class.

If we discovor the qualities upon which a class is based by direct observation and induction, by the same method we discover the definition of it.

§ 2. What is meant by a definition? Why definition is necessary; what is its place in Logic?—To define a name is to give a precise statement of its meaning or connotation. The name to be defined is the subject of a proposition, whose predicate is a list of the fundamental qualities common to the things or processes which the subject denotes, and on account of possessing which qualities, this name is given to them.

As classification enables us to understand the facts of Nature in a better way, so definition gives us also better understanding. What we find to be true in some cases we want to assert that that is also true in all similar cases *i. e.* in all cases belonging to the same class. But how should we know whether some particular cases belong to any particular class or not. We must know the essential characters indicated by the class-name and we must examine whether these particular cases possess

those characters. Every definition is nothing but the statement of essential characters possessed by a class of things. Mill says :—The purpose of definition is not to expound a name but a classification.

To form a good definition is a work of no small difficulty and one calling for no small sagacity. It involves careful observation, comparison and analysis of the things observed, abstraction of the mind from their differences and generalisation, besides the power of distinguishing primary from derivative qualities. In short the definition is the perfecting and completion of the process of conception. Definition is the logical instrument of the first division of the science, that relating to *Terms*. It remedies their indistinctness, by giving a precise and fixed meaning to every name capable of having such a meaning assigned to it, so that we may know precisely what attributes it connotes and what objects it denotes. It is only in this way that our assertion can have a fixed and determinate import. So its value can be easily estimated. Definition is essentially practical and is therefore a part of Applied Logic.

§ 3. The limits of definition : What names cannot be defined :—The limits of definition are two-fold : (*a*) A name whose meaning cannot be analysed cannot be defined. *So the names of our simple feelings cannot be defined*, because their meaning cannot be analysed. Resistance and weight, colour and its modes, many names of sounds, tastes, smells, heat and cold—in fact whatever stands for an unanalysable perception, cannot be made intelligible to any one who has not had experience of the facts denoted : they cannot be defined but only exemplified. A sort of genetic definition may perhaps

be attempted, as, if we say that colour is the special sensation of the retina, or that blue is the sensation produced by a ray of light vibrating about 700, 000 000 000 000 times a second; but such expressions can give no notion of our meaning to a blind man or to any one who has never seen a blue object.

Abstract names may be defined by defining the corresponding concrete: the definition of "human nature" is the same as of "man." But if corresponding concrete be a simple sensation (blue), this being indefinable, this abstract (blueness) is also indefinable.

(*b*) The second limit of definition is the impossibility of exhausting infinity, which would be necessary in order to convey the meaning of the name of any. individual thing or person. *Proper names cannot be defined.* In attempting to define a proper name if we stop short of infinity, our list of qualities or properties may possibly be found in two individuals; and then it becomes the definition of a class name or general name, however small the actual class. Hence we can only give a description of that which a proper name denotes, enumerating enough of its properties to distinguish it from everything else as far as our knowledge goes.

§ 4. **Rules of Definition**:—The following rules must be observed in framing a good definition:

I. *It should contain neither more nor less than the connotation of the term defined.*

II. *It should be clearer than the term defined, and should not, therefore, be expressed in unfamiliar, figurative or ambiguous language.*

III. *It should not consist of a term synonymous with that defined.*

IV. *It should never be negative when it can be affirmative*

Or to sum the rules into one,

A definition should be (i) adequate. precise, and (ii) clear, and should not be (iii) tautologous or (iv) negative.

§ 5. **Perfect and Imperfect Definition** :—A perfect or complete definition is one which expresses the whole connotation of the name; an imperfect definition does not express the whole connotation.

Imperfect definitions are divided into two classes :—(i) Incomplete definitions and (ii) Accidental definitions or Descriptions.

Incomplete Definition :—When a name is defined by *part only* of its connotation, we have an Incomplete Definition.

Incomplete Definations serve the practical purpose of Definitions when it happens that all objects which possess the enumerated attributes possess those also which are omitted.

Accidental Definitions or Descriptions :—They are definitions composed of any attribute or combination of attributes which (though they are not connoted by the name defined) happen to be common to the whole of the subject and peculiar to it. In this kind of definition the name of a class is defined by any of its inseparable accidents. It has been rejected from the rank of genuine definition by all logicians and has been termed description.

Accidental definitions may be raised to the

rank of incomplete or even of complite definition, it the definition (or description) thus formed, be convertible with the *name* which it professes to define; that is if it be exactly co-extensive with it, being predicable of everything of which it is predicable, and of nothing of which it is not predicable. Examples :—(i) Man is a mammiferous animal having two hands. (2) Man is an animal who cooks his food.

§ 6. **Definition by Type** :—A definition by Type is the act of pointing out a member of the class in question and saying "I mean something like that."

Instead of trusting to the resources of language and endeavouring to supply the deficiency in respect of a single word or notion by an appeal to adjacent words or notions, like all other kinds of definitions or descriptions, the definition by Type appeals to some kinds of sensible. It says practically, "I will show you what I mean by the name in question : I mean something like that," pointing to an object before us.

The value of such Definitions :—(i) The process involved in such definitions is really the same as that by which the child acquires knowledge and by which we are all apt to acquire about things (*a*) very unfamiliar to us or (*b*) so simple in their nature that it would be difficult to refer them to any higher genus:

(ii) We would not for a moment suggest that such a resource is not most serviceable in many cases—in many cases it is the only resource available,—but we must not forget that from the logical point of view, it is less of a true definition than is the so-called Description.

§ 7. Definition per Genus et Differentiam:—It is the statement of the genus and differentia of the name. In mentioning the *genus* we use a term which implies all the attributes common to the *species* whose name is the term to be defined and to all other co-ordinate species of that genus; and by adding the differentia, we complete the statement of the connotation by giving those attributes which differentiate that species from all such co-ordinate species.

A definition *per genus et defferentiam* assumes that the meaning of the name of the genus is known; but such an assumption is necessary to the science of Logic, which must regard the requirement of a definition of any particular term as an exception to the general rule that men are acquainted with the meaning of every term they use.

Mill's view on the doctrine that Definition should be per genus et differentiam:—(i) A definition per genus et differentiam is an incomplete definition. A definition should be *per genus et differentias*; as in that case only we may get a complete definition.

(ii) It is impossible thus to define all names capable of being defined,—*summum genera* for example.

(iii) The object aimed at by those who laid down this rule is unattainable. In the same manner in which a special or technical definition has for its object to expound the artificial classification out of which it grows, the Artistotelian logicians seem to have imagined that it was also the business of ordinary definition to expound the ordinary and what they deemed the natural classification of things, namely the division of them into two kinds; and to show the place which each kind occupies

as superior, collateral or subordinate among other kinds. But this is impossible.

Mill's view that a definition should be per genus et differentias and not differentiam criticised :—It must be remembered that when definition *per genus et defferentiam* is spoken of, it is not meant to imply that the differentia is a single attribute; it may be a group of attributes. Each species can have but one differentia *i.e.* one set of attributes to distinguish it from the co-ordinate species—when referred to any one particular genus. Hence it is inaccurate to speak, as Mill suggests, of definition *per genus et differentias*.

§ 8. Substantial and Genetic or Constructive Definition :—A *substantial Definition* is that in which the essential attributes of the class are enumerated as they exist in the complete concept. A *genetic or constructive Definition* is that in which a process is indicated by which the essential attributes of the name may be secured. The genetic definition is not a statement of the way in which the concept corresponding to the name *has been* formed in the mind, but of the way in which, by indirect means, we *may* form a concept or a mental picture of the motion, when it is inconvenient to say directly what it is. Genetic definitions are to be found in Mathematics.

§ 9. Analytically formed and Synthetically formed Definition :—An analytically formed definition expresses the ordinary meaning of a term. A synthetically formed definition gives a new meaning; it is the giving a new and arbitrary meaning to an old term, or the equally arbitrary

fixing of the connotation of a newly invented term, to serve the purposes of some special discussion.

Synthetically formed Definitions when legitimate :—Such definitions can only be regarded as legitimate when a new technical term is absolutely necessary in a science; then it is far better to invent a new term than to give a new and arbitrary meaning to an old one; for in the latter case, every one is apt to revert more or less unconsciously, to the ordinary signification of the term.

§ 10. Special or Technical Definitions :—In special languages and in special subjects, there are words peculiar to them—notions unfamiliar or unknown to the bulk of speakers outside its range—familiar notions new significations attached to them or entirely new creations. These words are called special or technical words and the statements of their meaning as accepted in special languages and subjects may be regarded as Special or Technical definitions.

Such definitions may be easily formed. The work of defining these special words is easier, and the definitions are more accurate than in the case of more generally familiar words. This is merely on the ground that a word confined to a special class is much more likely to retain a uniform and fully recognised signification than one which has to do duty over a very wide area.

§ 11. Nominal and Real Definition :—According to the Scholastic Logicians a *Nominal Definition* is one which unfolds the *meaning of a word*; and a *Real Definition* is one which explains the *nature of a thing*.

Mill holds that all definitions are of names and of names

only; but in some definitions it is clearly apparent that nothing is intended except to explain the meaning of the word; while in others besides explaining the meaning, it is intended to be implied that there exists a thing, corresponding the word. Definitions which merely declare the meaning of a name, without any assumption as to real existence of the subject, are called *Nominal Definitions.* Definitions which explain the meaning of a name, but at the same time assume the real existence of the subject, are called *Real Definitions.*

Is such a distinction necessary in Logic? It is better to discard this distinction from Logic, for it simply tends to confuse the whole object of logical definition by importing into it considerations with which the process of framing a definition is not rightly concerned.

VIII.

HYPOTHESIS AND EXPLANATION.

PART I.—*Hypothesis.*

§ 1. **What is Hypothesis?**—The word "Hypothesis" is derived from the two Greek words ὑπο *under*, and θέσις *placing*, and is therefore exactly synonymous with the Latin word *suppositio*, a placing under, whence our common word 'supposition.' It appears to mean in science the imagining of some thing, force or cause, which underlies the phenomena we are examining, and is the agent in their production without admitting of direct observation

§ 2. **Mill's definition**:—An hypothesis is any supposition which we make (either without actual evidence or an evidence avowedly insufficient) in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real; under the idea that if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be or at least is likely to be true.

§ 3. **Bosanquet's definition**:—"Hypothesis is a name that may be applied to any conception by which the mind establishes relations between data of testimony, of perception or of sense, so long as that conception is one among many alternative possibilities and is not referred to reality as a fact." Every supposition we make so as to account for any event whatever is a hypothesis.

§ 4. **The three stages through which a hypothesis passes** :—(i) the stage of mere conjecture (Gaseous stage); (ii) conjecture seriously put forward *i. e.* it is put forth as an explanation (Liquid stage); (iii) conjecture transformed into an established law (Solid stage).

§ 5. **How are hypotheses suggested** :—They are suggested by (i) Enumerative Induction, (ii) Conversion of Propositions and (iii) Analogy.

(i) **By Enumerative Induction** :—Every observed regularity of connection between phenomena suggests a question as to whether it is universal. Thus to take a simple example from Mathematics,—it is easily seen by simple inspection that $1+3=2^2, 1+3+5=3^2$, and so on. This suggests the hypothesis that in every case the sum of the first n odd numbers will be equal to n^2. But such a hypothesis can never be more than an empirical law *i. e.* a description of what relation actually does hold—until its necessity is established by a consideration of the essential properties of numbers, and this takes us beyond mere enumeration.

(ii) **By Conversion of propositions** :—Wherever a universal connection is established between phenomena, we are led to enquire whether it is reciprocal, and this again is an enquiry which can only be answered by such an analysis of content as will bring to light the exact grounds of the connection. Thus every conversion of an *A* proposition suggests a hypothesis.

(iii) **By Analogy** :—Analogy is the chief source from which new hypotheses are drawn. A relation which is already familiar in one class of phenomena suggests the direction in

which to look for the explanation of a new set of phenomena which bear some resemblance to the former. "A successful hypothesis is always due to the attention paid to analogies." (Lotze).

§ 6. **The objects of framing hypotheses** :—We may have the following objects of framing hypotheses ;—(i) we may desire to account for a particular phenomenon ; (ii) we may seek to establish a general law.

§ 7. **Mill's classification of hypotheses** :—There are three classes of hypotheses,—(i) Where the phenomenon assigned as cause is real, but the law according to which it acts, merely supposed. Where the cause is known but the law is not known. Example :—The motion of planets is determined by the attraction of the sun. We do not know that the attraction varies as the square of the distance. (ii) Where the cause is hypothetical, but supposed to operate under known laws *i. e.* supposed to produce its effects according to laws similar to those of some known phenomena. Example :—The Undulating theory, (iii) Where both the agent or cause and the law in operation are hypothetical. There is no end of this class of hypotheses.

§ 8. **Different kinds of Hypotheses** :—The object of making supposition varies, so there are different kinds of hypotheses :—(i) Constructive Hypothesis.

(ii) Descriptive Hypothesis.

(iii) Illustrative Hypothesis.

(iv) Practical Hypothesis.

(v) Deductive Hypothesis.

(vi) Hypothesis as method of Inductive Proof.

Constructive Hypothesis :—A supposition made in order to account for a given fact is called a Constructive Hypothesis. In order to detect a murderer, a detective forms a series of conjectures *i. e.* constructive hypotheses. He begins to think over the various possible agents. He says to himself :—"If it was A who did it, he must have been away from his work at the time: but he is known on the contrary to have been in the shop all day."—"If it was B, he must have got possession of a gun; but how and what has he done with it since?"—"If it was C. he would have been suspected at once and his appearance on the scene would have caused alarm."—"If however it was D, there seems no such difficulty to be accounted for." In this way we may proceed successively through a string of such suppositions.

Descriptive Hypothesis :—A supposition which is merely intended to describe a certain class of phenomenon. Another name of "Descriptive hypothesis" is "Representative fiction." When we talk of Electric fluid, we have a descriptive hypothesis. There is no such thing as electric fluid, but the phenomenon is like so. It is a kind of analogy or simile.

Illustrative Hypothesis :—We find writers on mathematical Physics or on Political Economy postulating special facts and not only these but also general laws, such as, we know for certain, never did happen and never will. In some works on Political Economy, in the course of a discussion about currency, we may have the question proposed as to what would happen, were the amount of coin in circulation to be suddenly doubled or halved. Such suppositions are proposed or designed for some scientific purpose. That purpose is

illustrative. We assume a supposition and deduce the results to test our law. The practical and constructive aim is not of course entirely absent here, but it is very remote and is quite subordinate to the speculative.

Practical Hypothesis :—The object of this is not to illustrate any law but it is preparatory to do something. (A chess-player forms such a hypothesis when he makes up his mind to make a certain move.)

Deductive Hypothesis :—Mill says, Geometrical truths are based on hypothesis. There is nothing in nature corresponding to the definitions of Euclid. Therefore the conclusions which we reach by Geometrical proof are all hypothetical. The hypotheses formed for the purpose of drawing deductive inferences from them are called Deductive Hypotheses.

(**Hypothesis as method of Inductive Proof** :—A hypothesis is established when it is proved to be the only one which will explain the facts. Hypotheses supply us with an indirect method of proof. When a hypothesis is rejected, another hypothesis is taken. In this way we come to the true hypothesis.)

§ 9. **Hypothesis and Theory** :—A theory is an established hypothesis. We should also remember that the word "theory" is highly ambiguous, being sometimes used as equivalent to hypothesis, at other times as equivalent to general law or truth.

§ 10. **Hypothesis established** :—When may a hypothesis be regarded as fully established ? In order to make out that your supposition is true, it is necessary to show, not

merely that a particular supposition will explain the facts, but also that no other one will. In other words, it must be possible not only to reason from the hypothesis to the facts, but from the facts back to the hypothesis, without any other supposition being possible at any step.

§ 11. **Crucial Instance; Experimentum Crucis** :—It happens sometimes that two or even more different hypotheses will explain a great number of the phenomena in question. In such a case, it is necessary to look for some instance which can be explained on one only of these rival hypotheses. As Ueberweg puts it :—"One single circumstance which admits of one explanation only, is more decisive than a hundred others which agree in all ponits with one's own hypothesis, but are equally well explained on an opposite hypothesis." Such a test-case is called a **Crucial instauce** and if arrived at by experiment is named **Experimentum Crucis.** The term is adopted from Bacon, who tells us that it is transferred from the crosses or finger-posts which are put up in crossings to mark and point out different ways.

§ 12. **Conditions that a legitimate hypothesis must conform to** :—

(i) It must not be gratuitous or unnecessary. Fowler says that a hypothesis may be gratuitous and yet true. But that is erroneously supposed to be gratuitous. Hamilton has stated the **Law of Parsimony** :—Nature never works by more and more complex instruments than are necessary. Mill says that he may reject gratuitous hypotheses without accepting the Law of Parsimony. If the supposition is known to be the

result of any known law, then the supposition of any other law is gratuitous or unnecessary.

(ii) A hypothesis must not conflict with any laws accepted as true on superior evidence,—laws of thought and laws of motion &c. A hypothesis which is conflicting with the primary laws of thought must be rejected.

(iii) It must be determinate so that we may be able to deduce from it the precise consequences, that will result, supposing it to be true.

(iv). The consequences deduced from it must agree with the facts of observation. As the very aim of every hypothesis is to express the relations which exist in reality, it is obviously essential that the hypothesis should be verified by comparison of the results deduced from it with facts of observation. In order that this may be done, it is essential that the consequences of the hypothesis should be inferred with the utmost precision and the comparison of those consequences with the facts should be made with great care and accuracy. The hypothesis must of course, in the first place agree with the phenomena, it was invented to explain. But we must not rest satisfied with this. It must be compared with facts of the greatest possible variety and exemplifying every possible case which can be brought under the hypothesis. A single absolute disagreement with facts is fatal to a hypothesis.

§ 13. Hypothesis non fingo :—Hypotheses are always useful. Wrong hypotheses generally precede right ones. Every hypothesis should be clearly apprehended and held subject to revision. What did then Newton mean when he said: "hypotheses non fingo?" It was against the assumption of hypo-

theses on little or no evidence, and obstinate adherence to them, that Newton protested when he said :—"hypothesis non fingo." He did not mean that he deprived himself of the facilities of investigation afforded by *assuming* in the first instance what he hoped ultimately to be able to prove. Without such assumptions, science could never have attained its present state ; they are necessary steps in the progress to something more certain ; and nearly everything which is now *theory* was once *hypothesis*.

§ 14. Relation between Hypothesis and Induction :—According to Whewell and Jevons, Hypothesis is an important stage through which we pass in our progress towards Inductive conclusion. It is a part of inductive reasoning. Mill says :—a hypothesis is a supposition formed without evidence or without sufficient evidence ; while an induction is not so. It is better to say that in hypothesis our evidence is indirect but in induction it is direct. In Induction, according to Whewell we *superinduce ideas upon facts*. This is practically the same as constructing a hypothesis, and testing it by facts. Thus, Hypothesis and Induction are different stages in the same process of reasoning. Jevons, according to whom, Induction is but Deduction inverted, seems to take the same view. Thus he thinks that all inductive investigation consists in forming hypothesis as to the causes of things and determining by experiment whether these hypothetical conditions are real causes or not.

§ 15. Hypothesis and Experiment :—Every hypothesis may be confirmed by Experiment. Every hypothesis suggests also how experiment is to be made. The one great

method of inductive investigation consists in the union of hypothesis and Experiment. When we consider this relation between hypothesis and Experiment, we classify our knowledge under four heads ;—

(1) We may be acquainted with facts or phenomena which have come under our notice accidentally or without reference to any special hypothesis and which have not been brought into accordance as yet with any hypothesis. Such facts constitute what is called simply *Empirical knowledge.*

(2) Another very extensive portion of our knowledge consists of those facts which, having been first observed empirically, have afterwards been brought into accordance wiih other facts by an hopothesis concerning the general laws applying to them. This portion of our knowledge may be said to be *explained, reasoned* or *generalised.*

(3) In a third place comes the collection of facts, minor in number, but most important as regards their scientific value and interest, which have been anticipated by hypothesis and afterwards verified by experiment.

(4) Lastly there may and does exist knowledge of phenomena accepted solely on the ground of hypothesis and which is incapable of experimental confirmation, at least with the instrumental means at the time of our possession.

§ 16. Relation between the Constructive hypothesis and Explanation :—A constructive hypothesis is framed to explain phenomena not explained, *i.e.* whose cause is unknown.

§ 17. Relation between Hypothesis and Deductive Reasoning :—In a deductive Reasoning. we pass

from a general law. Hypotheses are invented to enable the Deductive reasoning to be earlier applied to phenomena. In every deductive method we want to determine complex laws by the help of simpler laws. In hypothesis complex laws are known, we want to establish simple laws.

§ 18. Hypothesis, Induction and Deductive Reasoning :—There is a doctrine that the hypothesis of the Deductive Reasoning is the inference of the Inductive process. The special facts which are the conclusions of Deduction are the basis of the Inductive Inference.

PART II.—*Explanation.*

§ 1. Explanation; Its ordinary sense :—In the ordinary use of the word, *Explanation* means the satisfying a man's understanding; and what may serve this purpose depends (i) partly upon the natural soundness of his understanding, and (ii) partly on his education.

Popularly an *explanation* means the substitution of a mystery which has become familiar, and so ceased to seem mysterious, for one to which we are still unaccustomed.

§ 2. Definition of Explanation in Logic :—An *individual fact* is said to be explained by pointing out its cause *i.e.* by stating the law of causation of which its production is an instance.

A law or uniformity is said to be explained when another law is pointed out, of which the law is itself but a result, and from which it may be deductively inferred.

§ 3. Nature of Explanation :—When the conditions of every detail of a phenomenon are so fully and exactly known, that not only a phenomenon of that general character, but just this very phenomenon, with exactly these details, and each in exactly this amount, must follow from those conditions and from those only, then that phenomenon is fully explained. In the vast majority of cases such thoroughness of explanation is not attained, and is not even sought. It is some particular aspect or aspects of the whole complex particular fact that it is desired to explain, and much of the rest of the detail is passed over as having no bearing on the purpose in hand.

§ 4. Popular and scientific Explanation contrasted :—In a popular explanation we substitute what is familiar for what is not familiar, or what is more familiar for what is less familiar. In a philosophical explanation our business may not be so ;—may be precisely the reverse of this. It may resolve a phenomenon with which we are familiar into one of which we previously knew little or nothing ; as for instance when the familiar law "All bodies tend to fall to the earth," was found to be a case of the previously unknown law,—"Every particle of matter attracts every other."

§ 5. Mill and Bain distinguish three principal modes of scientific Explanation.

(1) The Analysis of a phenomenon into the laws of its causes. It is by resolving the law of a compound effect into the separate laws of the concurrent causes, and the fact of the co-existence of those causes.

(2) The discovery of steps of causation between a cause

and its remote effect. It is by detecting an intermediate link in the sequence of causation.

(3) The subsumption of several laws under one more general expression.

§ 6. **Observations on the three modes of Explanation :—**

(1) The first is the case of the composition of causes producing a joint-effect equal to the sum of the separate effects. The explanation of such an effect involves two things :—(i) the simpler laws of the separate causes; and (ii) the fact of the co-existence of those causes (for if not co-existent they could not intermix their effects).

Example :—Why does A B produces XY ?

(i) ∵ A produces X ;
and B produces Y ;
(ii) A and B here act together
∴ A B produces XY.

(2) The second mode of explaining a law is to point out an intermediate link between an effect and its assigned cause, to show that this assigned cause is only the cause of the cause.

Example :—A produces C ; how do you explain it ?

∵ A produces B
and B produces C
∴ A produces C.

(3) The third mode is the subsumption of less general laws into a more general one ; *i.e.*, the less general laws are found to be merely instances of the operation of the more general.

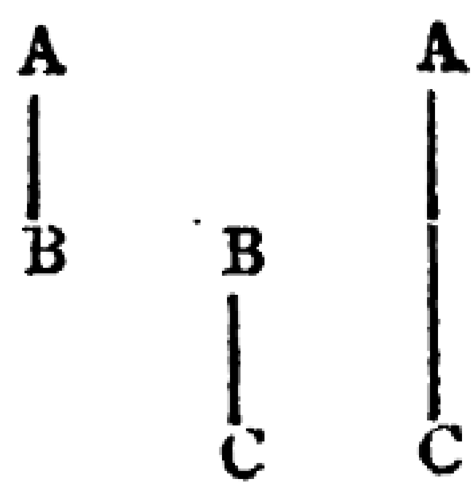
Example :—Why do heavy bodies fall to the earth ?

∵ There is the Law of Gravitation.

§ 7. When any law is explained, more general laws are stated.

(1) In the first kind of explanation it is evident that each of the laws, "A produces X," and "B produces Y" is more general than the law AB produces XY" ; because we get this last law only when A and B act together. But A and B may exist together,—may exist also separately. When they exist separately, then also we find the operation of the two laws "A produces X," and "B produces Y" ; when A and B co-operate, then also those laws are operative. Consequently these two laws are more general than the law explained.

(2) In the second kind of explanation also the statement is true.



The law that A produces C is explained by two other laws:—A produces B, and B produces C. When A produces C, then A first produces B and afterwards B produces C. But there may be cases where A produces B, but C is not produced by B on account of some other counter-acting cause; so "A produces B" is more general than "A produces C." Again there may be cases where B is not produced by A but by some other cause, but still produces C; so "B produces C" is also more general than "A produces C."

(3) In the third kind of explanation a more general law is necessarily to be mentioned.

§ 8. **The Limits of Explanation** :—As Explanation consists essentially in deriving laws from higher ones by showing that they all agree in being applications of one common law under different sets of circumstances, it follows that we may go on explaining the lower by the higher until we come to laws of uniformities so high and general that they have nothing in common and cannot be reduced to anything higher and more general,—in other words to ultimate laws. Thus no one has ever succeeded in explaining the law of gravitation. Thus the limits of Explanation are the limits of Induction;—Explanation must stop where Induction has reached the highest stage of generalisation.

§ 9. **All Explanation is relative** :—All explanation may be regarded as relative, because in explaining a law we reach at last to a law or laws unexplained.

§ 10. **Explanation; Ultimate Laws and Derivative Laws** :—The Ultimate Laws are not to be explained, but the Derivative Laws are to be explained.

The *Ultimate Laws* are those which are not deducible from or resolvable into other and more general laws, however much our knowledge be extended.

The *Derivative Laws* are those which are deducible from or resolvable into other and more general laws.

The ultimate Laws of Nature cannot possibly be less numerous than the distinguishable sensations or other feelings of our nature—that is those feelings which are distinguishable in *kind* or *quality* and not merely in degree.

As long as a law is known to be ultimate, we cannot explain it. But a supposed ultimate law can be explained, when it transpires to be Derivative

§ 11. **An Empirical Law may be explained.**

We must remember that the expression "Empirical Law" has been used in two different senses by Mill and others :—

(i) An unresolved derivative law *i. e.* a law capable of explanation but which we do not yet know how to explain.

(ii) Any law based on Induction by simple enumeration without admixture of deductive reasoning *i. e.* unexplained law whether ultimate or not.

Now we must take it in its first sense. As an Empirical Law is an observed uniformity, presumed to be resolvable into simpler laws, but not yet resolved into them, or a law whose why has not been ascertained, it is really a derivative law ; so it may be explained.

§ 12. **Hypothesis, Induction and Explanation.**:—We frame hypothesis in order to reach explanation, when it does not appear to us at once. We frame hypothesis to explain phenomena otherwise unexplained. Every Hypothesis is an attempt at explanation ; every established theory is an explanation of greater or less scope.

All Induction is explanation as every Explanation consists essentially in detecting the universal in the particular *i. e.* in deducing or deriving the particular event from, or showing it to be an application of some universal law. But this involves generalisation,—rising from the multiplicity of particular facts to some general law which would include

and *explain* them. Thus there is generalisation both in Induction and Explanation.

§ 13. **Proof and Explanation of a Law** :—Are they one and the same ? The answer depends on the consideration of the qnestion whether the proof is Deductive or Inductive. When it is deductive, the proof and the Explanation of a law are the same. When the law is established by direct induction, then they are not the same. In this case proof is independent of Explanation.

§ 14. **Fallacious and Illusory Explantions** :— Dr. Bain has mentioned the following three types of fallacious and illusory explanations :—

(1) One form of illusory explanation is *to repeat the fact in different language*, assigning no other distinct and yet parallel fact.

Examples :—(i) Why does opium produce sleep ? ∵ It is a sleep-producing thing. (ii) Why must the future resemble the past ? ∵ Nature is uniform.

(2) Another illusion consists in *regarding phenomena as simple because they are familiar*.

Very familiar facts seem to stand in no need of explanation themselves, and to be the means of explaining whatever can be assimilated to them.

Examples :—(i) The lighting of a fire, by contact with a flame, is a great scientific difficulty ; yet few people think it so. (ii) Voluntary action, from familiarity, has long been reckoned so simple in itself as to have provided a satisfactory explanation of all other modes of generating mechanical force.

(3) The greatest fallacy is *the supposition that something*

is to be desired beyond the most generalised conjunctions or sequences of phenomena.

Examples:—(i) The union of Body and Mind has long been considered the mystery by pre-eminence. The prevailing opinion has been that this connection would for ever resist and paralize explanation. The material properties and mental properties are each to be conceived according to their own nature—the one by the senses, the other by self-consciousness. We also endeavour to rise to the most general laws of the union of the two classes of properties in the human and animal organization. When we succeed in carrying this generalizing operation to the utmost length that the case appears to ad mit of, we shall give a scientific explanation of the relationship of Body and Mind. It we ask for any further explanation, that will be fallacious,

(ii) The acceptance of the law of universal gravitation as full and final solution of the problem of falling bodies is the proper scientific attitude of mind. If we want to explain again this law of univers al gravitation, that is fallacious.

IX.

INDUCTION.

§ 1. **Definitions of Induction** :—(i) Induction is that operation of the mind by which we infer that what is true in a particular case or cases, will be found true in all cases which resemble the former in certain assignable respects.

(ii) Induction is the process by which we conclude that what is true of certain individuals of a class is true of the whole class, or that what is true at certain times will be true in similar circumstances at all times.

§ 2. **Carveth Read's Definition of Induction** :—An Induction is an universal real proposition based on observation, in reliance on the uniformity of nature: when well ascertained, it is called a Law. Thus, that "All life depends on the presence of Oxygen," is (1) an universal proposition; (2) a real one, since the "presence of Oxygen" is not connoted by "life"; (3) it is based on observation; (4) it relies on the uniformity of nature, since all cases of life have not been examined.

§ 3. **Mill's special Definition** :—Induction is (*a*) an inference, (*b*) establishing a general proposition, (*c*) on the evidence of particular instances.

In order to understand this definition we must know what Mill means by (*a*) an inference, (*b*) a general proposition.

(*a*) An **Inference**, according to Mill, is a statement

containing an information not given by the premises, but obtained through them. It is wider than the total of the premises. The additional knowledge conveyed by an inference is not even implicitly involved in them. All inference must be from facts known to facts unknown. Mill says, a deductive conclusion is not really an inference, as the information conveyed by it is involved implicitly if not explicitly in the premises.

For Example :—All men are mortal.
Henry is is a man.
∴ Henry is mortal.

In this syllogism the conclusion does not give us any new information. Those who understand the premises know also the truth of the conclusion; so there is no additional knowledge. Therefore according to Mill, it is not an *inference*.

Now take an inductive argument :—

The man A is mortal.
The man B is mortal.
The man C is mortal.
∴ All men are mortal.

Here the conclusion conveys some new information, which was not given by the sum total of the premises. The premises inform us that only 3 persons are mortal; but the conclusion tells us that not only three men but all men are mortal.

NOTE :—We reserve now our criticism of this view of Inference.

(*b*) **A general proposition** is one in which a predicate is affirmed or denied of *an unlimited indefinite number* of individuals, namely, all whether few or many existing or

capable of existing in present, past or future, which possess the attributes connoted by the subject-name. Or it is one which asserts that one phenomenon always accompanies (*i. e.* is a mark of) another phenomenon.

Illustration:—"All continents possess large rivers" is not a true logical general proposition, but only a bundle of four singular propositions, *viz.*, (1) Europe possesses large rivers; (2) Asia possesses large rivers; (3) Africa possesses large rivers; and (4) America possesses large rivers. Nothing has been affirmed of an indefinite number of continents. We cannot properly say attributes connoted by "continent" are marks of attribute "possessing large rivers"; the two only happen to be associated in the only cases which we happen to know, but if a new continent were raised, say from the bottom of Pacific Ocean, we have no assurance that it would contain large rivers. "All the apostles were Jews" is not a general proposition, as we cannot predict from this, that if an apostle were at any future time appointed, he would be a "Jew,"—the attributes connoted by "apostles" are not marks of the attribute connoted by "Jew." "All men are mortal"—is a true general proposition.

§ 4. The position of Induction in Logic as mentioned by Mill:—"We have found that all Inference consequently all Proof and all discovery of truths not self-evident consists of inductions, and the interpretation of inductions: that all our knowledge, not intuitive, comes to us exclusively from that source. What Induction is, therefore, and what conditions render it legitimate, cannot but be

deemed the main question of the science of Logic—the question which includes all others."—Mill.

§ 5. The Different stages of Inductive investigation :

(1) We first of all observe some facts and then form a *hypothesis* regarding all such facts. The first stage is then the formation of hypothesis after observation of facts.

(2) We apply the Experimental Methods and then come to a generalisation *i. e.* an Inductive conclusion. The second stage then is the conclusion proper after application of methods.

(3) Verification.

§ 6. There are some processes of reasoning which are improperly called Inductions. Mill mentions the following :—

(1) Perfect Induction.

(2) Some reasoning used in Mathematics.

(3) Inferences by Parity of Reasoning.

(4) Mathematical Induction.

(5) Colligation of Facts by an appropriate conception.

§ 7. Perfect Induction :—The Scholastic Logicians distinguished Induction as Perfect or Imperfect. When any statement is made regarding a class of objects, after examining each and every member of the class, then the Induction, according to them, is called **Perfect** ; and when any statement is made regarding a class of objects, after examining some members only, then the Induction is called **Imperfect**. If we examine the case of Peter, of Andrew, of John, of James, of Philip, of Barthol mew, of Matthew, of Thomas, of James

the son of Alphaeus, of Thaddaeus, of Simon the Cannanite, of Judas Iscariot and of Paul separately and come to the conclusion that "All the Apostles are Jews,"—it will be a Perfect Induction according to the Scholastic Logicians. But if we examine the cases of three men A, B and C and come to the conclusion that "All men are mortal,"—it will be an Imperfect Induction according to them.

Mill is right in thinking that the so-called Perfect Induction is not Induction at all, inasmuch as (1) there is no inference from facts known to facts unknown,—any operation involving no inference in Mill's sense of the term, any process in which what seems the conclusion is no wider than the premises from which it is drawn, does not fall within the meaning of the term Induction; (2) The conclusion of the so-called Perfect Induction is not a General Proposition;—it is only the summation of the premises.) Induction proper is what is known to the Scholastic Logicians, as Imperfect Induction.

§ 8. Some reasoning used in Mathematics:— Mill says that there are several processes used in Mathematics which require to be distinguished from Induction, being not unfrequently called by that name, and being so far similar to Induction properly so called, that the propositions they lead to are really general propositions. For example the following propositions, are first separately proved:

(1) A straight line cannot cut a circle in more than two points.

(2) A straight line cannot cut an elipse in more than two points.

(3) A straight line cannot cut a parabola in more than two points.

(4) A straight line cannot cut a hyperbola in more than two points.

We then conclude that "a straight line cannot cut a conic section in more than two points." We know that a cone is intersected by a plane in one of these ways. There is no Induction here, as there is no real inference. The conclusion is a mere summing up of what was asserted in the given propositions from which it is drawn. This may be regarded as a case of the so-called Perfect Induction.

Mill has regarded the conclusion of the so-called Perfect Induction as *general*. But Lotze calls it *universal*, not *general*. When we predicate something of a whole *indefinite* class, the statement according to Lotze is *General*; but when we predicate something of a whole *definite* class, it is *Universal* according to him.

§ 9. **Inferences by Parity of Reasoning**:—When we prove a Geometrical theorem by means of a diagram, we have this kind of reasoning. Having shewn that the three angles of the triangle A B C are together equal to two right angles, we conclude that this is true of every trangle not because it is true of A B C, but for the same reason which proved it to be true of A B C. This is called an Inferance by Parity of Reasoning, *i. e.* owing *to the sameness of the reasoning* that may be applied to all other cases.

If this were to be called Induction, an appropriate name for it would be *Induction by Parity of Reasoning*. But the term cannot belong to it; the characteristic quality of Induc-

tion is wanting; (since the truth obtained though really general, is not believed on the evidence of particular instances.)

If we come to the truth by measuring the three angles of several triangles equal to two right angles, then it is an Induction. But we do not do that; we really draw a diagram of a particular triangle as a representative or symbol of triangles.

Mill says,—"we prove something in a particular case and imagine the same proof applied to the other cases; so an inference by Parity of Reasoning is only the summation of several conclusions,—we gather up into one general expression all the singular propositions susceptible of being thus proved". This statement of Mill is not good; there is no summation in it. We get this inference, not by summing up several singular propositions but because we perceive that the similar proof might be applied to the other cases.

§ 10. Mathematical Induction or Induction by Connexion :—(A mathematician) when he has calculated a sufficient number of the terms of an algebraical or arithmetical series to have ascertained what is called the *law* of the series, (dose not hesitate to fill up any number of the succeeding terms without repeating the calculations. Such a process of reasoning is generally called Mathematical Induction.)

Illustration :— $1+3=4=2^2$

$1+3+5=9=3^2$

∴ The sum of any number of odd numbers beginning with 1 is the square of that number.

According to Mill, such a conclusion is drawn when it is apparent from *a priori* considarations, (which might be

exhibited in the form of demonstration) that the mode of formation of the subsequent terms, each from that which precedes it, must be similar to the formation of the terms which have been already calculated. And when the attempt has been hazarded without the sanction of such general considerations there are instances on record in which it has led to false results.

The so-called (Mathematical Induction may be regarded as a particular case of Induction by Parity of Reasoning) It is not an inference of a general proposition from particular instances ; so it is improperly called Induction.

§ 11. Colligation of Facts by an appropriate conception :—What is *colligation ?* We are said to *colligate* a number of phenomena when we unite them into a single class by forming a *general notion* of their *Common attributes.* These Common attributes may include both the essential and the accidental qualities, and when these are expressed in words, we have a *description* of them. Thus colligation is the mental process by which we bring a number of phenomena under a *general description.*

(Induction has been defined by Dr. Whewell as the colligation of facts by an appropriate conception.) Dr. Whewell maintains that the general proposition which binds together the particular facts, and makes them as it were one fact, is not the mere sum of those facts, but something more, since there is introduced a conception of the mind which did not exist in the facts themselves.

Mill's View :—Induction and colligation are two distinct processes. Induction is something more than colliga-

tion. Colligation is not always Induction; but Induction is always colligation. A conception or hypothesis can describe facts; but unless it is proved or established, it cannot be regarded as an Inductive conclusion.

According to Mill (1) there is no real inference in mere colligation. (as in the statement under [illegible]. All men are

(2) Successive colligations, though conflicting, may yet all be correct, so far as they go; that is they may all correctly represent the facts observed at the time they were respectively framed. In different stages of the progress of knowledge, philosophers have employed, for the colligation of the same order of facts, different conceptions. Now, it would be absurd to assert that conflicting Inductions could all be true.

(3) Colligation only describes; Induction besides incidentally describing, also predicts and explains.

Criticism of Mill's view:—We cannot altogether agree with Mill in rejecting Whewell's statement that Induction is the colligation of facts by an appropriate conception. The conceptions by which facts are to be colligated are hypotheses. We must remember that we pass through three different stages, when we want to draw an inference by Inductive reasoning:—(1) We observe facts and form a hypothesis; (2) We apply the experimental methods and get the general proposition; and then (3) We verify the Inductive conclusion by bringing other particular facts belonging to the class but not examined before, and finding that the general truth holds good also in those cases. Dr. Whewell as a discoverer laid stress on the first stage of Inductive reasoning. According to him, the formation of an hypothesis is the main business of an Inductive reasoner.

When a hypothesis is obtained, the next business is only to establish it; so the other functions are only subsidiary to this main function. Mill gives importance to the second stage; and that is why he condemns Whewell's statement so much. According to him Induction is Proof; it is inferring something unobserved from something observed; it requires, therefore, an appropriate test of proof; and to provide that test of proof is the special purpose of Inductive Logic. It will not be altogether out of place to mention here that Whewell lays stress upon the first stage, Mill on the second stage; and Jevons lays stress on the third stage of Inductive reasoning when he defines Induction as the Inverse Process of Deduction.

§ 12. **Induction as the Inverse Process of Deduction**:—An "inverse process" presupposes a "direct process" with which it is connected in the following way. The *direct process* starts from certain data and reaches a conclusion. That is, the particular premises and the law according to which we are to infer, are supposed to be given whilst the conclusion is to be obtained by us. The *inverse process*, on the other hand, starts from a conclusion and tries back for the data. That is, the conclusion and the laws according to which we are to infer are supposed to be given, whilst the premises are to be obtained by us.

In a Deductive reasoning we pass from a general statement to particular facts,—from causes to effects. In an Inductive reasoning we pass from particular facts to a general statement,—from effects to causes; so it is called the Inverse Process of Deduction. In Induction, according to Jevons, we obtain our

law more or less conjecturally by direct experience and afterwards verify it.

Jevons thus states his view :—"All Inductive reasoning is but the inverse application of Deductive reasoning. (1) Being in possession of certain particular facts or events expressed in propositions (2) we imagine some general proposition expressing the existence of a law or cause (3) and deducing the particular results of that supposed general proposition (4) we observe whether they agree with the facts in question. Hypothesis is thus employed, consciously or unconsciously. The sole condition to which we need conform in framing any hypothesis is that we both have and exercise the power of inferring deductively from the hypothesis to the particular results, which are to be compared with known facts."

According to Jevons, thus there are but four steps in the process of Induction :

(1) Preliminary observation of particular facts.

(2) Framing of some hypothesis as to the character of a general law. (3) Deducing consequences from that law.

(4) Observing whether the consequences of that general law agree with particular facts under consideration.

So Jevons makes prominent the last stage of Inductive reasoning, namely, Verification. In Verification we have the deductive reasoning ;—after obtaining a general law, we examine whether that general law holds good in particular cases,—which we do by deducing consequences from the general law and comparing them with known facts. The previous part of our Inductive reasoning becoms complete when we reach this stage.

§ 13. The Old and Obsolete Views of Induction ?

(1) Aristotle's account of Induction :—

(i) Induction is ascending from the particular to the universal, in the sense of recognizing the universal in the particular .

(ii) It is a kind of syllogistic argument in the third figure.

Example :—

$s_1, s_2, s_3, \quad \ldots \quad \ldots \quad S_n$ are P

$s_1, s_2, s_3, \quad \ldots \quad \ldots \quad S_n$ are all S

∴ All S are P.

Example :—

Man, horse, mule &c are long-lived.

Man, horse, mule &c are bileless.

∴ Bileless animals are long-lived.

Criticism :—Aristotelian Induction is a Perfect Induction. It is difficult to obtain a complete enumeration of individuals.

Syllogisms in the third figure give us particular conclusions. ∴ It is not an ordinary syllogistic form.

(2) The View of the Scholastic Logicians :

The Scholastic Logicians made Induction entirely enumerative and distinguished the process as Perfect and Imperfect This doctrine was practically universal amongst logicians till the time of Bacon. Induction by Simple Enumeration is ascribing the character of a general truth to any proposition which happens to be true in every instance we have known of, *i. e.*, to which we do not heppen to know any exception. Bacon says, about Induction " per Enumerationem simplicem,"

"ubi non reperitur instantia contradictoria" *i. e.*, it is induction because we have never found an instance to the contrary.

Criticism :—It is an argument from simple unanalysed experience ; its formula being "such and such has always been found to be true ; no instance to the contrary has ever been met with ; ∴ such and such is true." (All crows hitherto observed have been black ; no crow of any other colour has ever been seen ; ∴ all crows are black. The conclusions of such an induction can never be taken as certain.) According to Bacon, the Induction which proceeds by simple enumeration is a childish matter and infers rashly and is always liable to be refuted by a negative instance.

§ 14. Empiricist View and Rational View regarding Inductive Reasoning.

Induction is the doctrine of those valid processes of thought by which knowledge of what holds universally in reality is attained. It is true that in experience alone we are brought into contact with reality. But what is Experince ? In answering this question the Empiricists and the Rationalists differ from each other.

Empiricist View :—By "experience," the Empiricists mean mere succession of sensuous impressions. The only reality for the Empiricists is the series of sensations in our individual consciousness. With objective reality goes, of course, objective or universal knowledge. A doctrine of Induction cannot, therefore, be consistently based on Empiricism.

Rational View :—According to the Rationalists, "experience" is not simply what is felt ; it is what is thought.

Experience is not a mere series of sensations, but it is the totality of those impressions organized and systematized by the constitutive activity of mind. A fact exists only in relation to other facts, and these relations only exist for thought, they are not open to sensuous observation. Every such relation is universal *i. e.* it is applicable not to this one fact because of its individuality, but to this one fact as a type or example of a class of facts. Whilst in sensuous impressions we have facts presented in their particular aspect, in thought we grasp their universal aspect. Experience which includes both impression and thought is thus seen to unite the universal and the particular.

Mills Position :—Mill adopted the fundamental empiricist position that nothing is given as a basis of knowledge but separate and particular sensations. The outcome of Empiricism is that knowledge is impossible. Mill attempted to avoid this conclusion by his doctrine of Induction. He defined Induction as "discovering and proving general propositions." Throughout Mill's Logic, we find two incompatible theories of inference—first that inference is based on resemblance between phenomena, and second, that inference is grounded on the essential conditions of phenomena. The former view is *empirical*, but the latter is *rational*. The former view is the foundation of his advocacy of the doctrine that "all inference is from particulars to particulars,"—that observing A, B and C's mortality we can at once infer D's mortality, who is also like A, B, C. This view is also expressed in several of Mill,s definitions of Induction. The second view is found in his actual treatment of Induction. There he tells us

that to determine the effect of every cause and the causes of all effects is the main business of Induction. The latter position is quite inconsistent with Empiricism.

X

ANALOGY AND PROBABILITY.

PART I.—*Analogy.*

§. 1. **The meaning of Analogy: What is Analogy ?**—One of the meanings of Analogy is *Resemblance of Relations.* Analogy is a kind of probable proof based upon imperfect similarity between the *data* of comparison and the subject of our inference.

§ 2. **Analogical Argument:**—Arguing from Analogy *generally* signifies to infer from resemblances in some points (not necessarily in relations), resemblance in others. Jevons says—"In Analogy, we reason from likeness in many points to likeness in other points." In the strict sense it is to infer from some resemblance of relations, resemblance in others.

Examples of an Analogical argument not used in its strict sense :—A certain object A, having a certain property *x*, has also a certain other property *m* ; another object B resembles A in possessing *x* ; ∴ B also resembles A in possessing *m*.

The earth possesses spherical shape, &c., and is also inhabited ; Venus resembles the Earth in possessing the former set of properties ; ∴ Venus is inhabited.

Example of an Analogical argument used in its strict sense :—The relation between a king and his people resem-

bles the relation between a father and his children. The children ought to obey their father ; ∴ The people ought to obey their king.)

§ 3. **Analogy, Deduction and Induction** :—(i) Like Deduction and Induction, Analogy assumes that things which are like in some respects, are also alike in others ; but (ii) it differs from them in not appealing to a *definite general law* assigning the essential points of resemblance upon which the argument lies.

In Deductive proof this is done by the major premise of every syllogism ; if the major says that " All fat men are humourists," and we can establish the minor " X is a fat man," we have secured the *essential* resemblance that carries the conclusion. In Analogy the essential resemblance is not so secured.

In Induction, the Law of Causation and its representaives,—the canons serve the same purpose, specifying the essential marks of a cause. But in Analogy, the resemblance relied on cannot be stated categorically. In Induction also, we infer from resemblance from some points, resemblance in others, but Analogy differs from Induction in not requiring the previous proof, by comparison of instances, of the invariable conjunction between the known and the unknown properties ; though it requires that the inferred properties should not have been *ascertained to be unconnected* with the common properties.

(**In Induction** the inferred properties and the common known properties must be *known to be connected*. In **Analogical reasoning** their *connections are not known ; neither are they known to be unconnected.*)

Another distinction :—" The Analogical argument is distinguished from Inductive one in this—that Induction regards a single predicate in many subjects, as the attribute *z* in A, in B, in C, in D, in E, in F, &c.; and as these may belong to one class, say Q, it is inferred that *z* will likewise be met with in the other things belonging to this class, that is, in all Qs. On the other hand, Analogy regards many attributes in one subject (Say *m*, *n*, *o*, *p* in A); and as these many are in part found in another subject (Say *m*, and *n* in B), it is concluded that, in that second thing, there will also be found the other attributes (Say *o* and *p*). Through Induction we, therefore, endeavour to prove that one character belongs (or does not belong) to all the things of a certain class, because it belongs (or does not belong) to many things of that class. Through Analogy, on the other hand, we seek to prove that all the characters of a thing belong (or do not belong) to another or several others, because many of these characters belong (or do not belong) to this other or those others. In the one it is proclaimed—" *One in many, therefore one in all.*" In the other it is proclaimed—" *Many in one, therefore all in one.*"—Hamilton.

§ 4. Analogical argument is only probable, and that in various degrees.

(1) The greater the number and importance of the points of agreement, the more probable is the inference.

(2) The greater the number and importance of the points of difference, the less probable is the inference.

(3) The greater the number of unknown properties in he subject of our agreement, the less is the value of any inerence from those that we know.

§ 5. An Analogical argument may come very near to a valid Induction, if

(i) The resemblance is very great.

(ii) The dissimilarity very small.

(iii) Our knowledge of the subject-matter tolerably extensive.

§ 6. Analogical reasoning useful. Why ?—The conclusions of analogy are not of direct use, unless when the case to which we reason is a case *adjacent in circumstances*, *i. e.* immensely similar. Even then a complete Induction should be sought after. But the great value of Analogy, even when faint, in science is that it may *suggest observations and experiments*, with a view to establishing positive scientific truths, for which, however, hypotheses based on analogies are required and must not be mistaken.

PART II.—*Probability*.

§ 1. What is meant by the word "Probable" in Logic ?—Usually when we say that an event is "*probable*," we mean that it is more likely than not to happen. But, in Logic, an event is "*probable*" if our expectation of its occurrence is less than certainty, as long as the event is not impossible.

§ 2. Why some facts are known as probable only ?—When in any particular case, we do not know what conditions are operative, we cannot tell, on the one hand, what result will appear, nor, on the other hand, can we say

positively what conditions have produced a certain given event. In such cases we are accustomed to speak of the occurrence as due to **Chance. Chance** was once believed to be a distinct power in the world disturbing the regularity of Nature. It is now admitted that every event in the world is due to some cause. Where our knowledge is complete, then the idea of chance would disappear ; it is due solely to the imperfection of that knowledge. Some facts are known as probable only because we do not know distinctly their operative causes.

§ 3. **The Theory of Probability in Logic** :—The probability of a given effect is the measure of *rational expectation* of its occurrence. Some facts are not certain but only probable to us owing to our imperfect knowledge of their operative conditions. The question then arises as to what we ought rationally to expect. The *estimation of such rational expectation* is the province of the theory of Probability. So it has a place in Logic.

§ 4. **The foundation of the Theory of Probability: How Probability is represented:** The foundation of the theory is that possibilities can be limited to a definite finite number. When we say that the probability of a thing's occuring in a certain way is $\frac{3}{7}$, this rests on the assumption that the seven possibilities are equally probable.

Probability is represented by a fraction. Taking I [one] to stand for certainty and o [zero] for impossibility, probability may be $\frac{999}{1000}$ or $\frac{11}{1000}$ or (generally) $\frac{1}{m}$. The denominator represents the number of times that an event happens and the

numerator the number of times that it coincides with another event *i. e.*, the number of occurrences in a particular way.

§ 5. **The Ground of Probability**:—It has already been mentioned,—when we say that the probability of a thing's occurring in a certain manner is $\frac{1}{7}$, there is an assumption that the seven probabilities are equally probable. Now the question is—what is meant by "equally probable"?—Or in other words, what is the ground of probability?—How do we know that there are so many possibilities?

The answer to this question is given from three different standpoints:—(i) Subjective, (ii) Objective and (iii) Subjectivo-Objective.

Subjective View:—According to this view, Probability depends upon the quantity of our Belief in the happening of a certain event or of its happening in a particular way. This is based on a psychological conception, *viz.*, that what is equally probable to an individual is that about which he feels equal confidence or expectation or belief. Such belief may have various psychological causes, such as personal disposition, temperament or mood; it is likely to be biased by feeling.

Objections against this View:—(*a*) Belief does not uniformly correspond with the state of the facts. (*b*) Belief cannot by itself be satisfactorily measured. (*c*) If Probability is to be connected with Inductive Logic, it ought surely to rest upon the same ground namely Experience. Induction, in any particular case is not content with beliefs or opinions, but aims at proving, testing, verifying or correcting them by appealing to the facts; and Probability has the same object

and the same basis. So this view must be excluded from logical consideration.

Objective View:—This view is based on actually experienced frequency. This objective frequency may be ascertained by statistics or by Deductive arguments. Such knowledge is open to everybody.

Subjectivo-Objective View:—The Subjectivo-Objective view takes account of the individual's knowledge,—not of his mere belief. It recognises that to different individuals the probability of the same event may be different: not because they feel a different degree of confidence, but because their knowledge of the case may be different.

§ 6. Direct and Inverse Probability:—In *direct probability*, we seek to determine the probability of an event being of a certain character, when it is happened under given conditions. E. G.—What is the chance of a white ball being drawn from a bag which contains 6 white and 4 black balls? Answer:—$\frac{3}{5}$.

In *Inverse Probability*, our position is reversed. Given an event of a certain character, we seek to determine the probability of its having resulted from any paritcular set of circumstances under which it might occur. E. G.—A white ball has been drawn from a bag containing 10 balls. What is the chance of the bag's having contained 6 white balls?

The following hypotheses are equally possible.

1. All the balls white.
2. 9 balls white, 1 black.
3. 8 balls white, 2 black.
4. 7 balls white, 3 black.

5. 6 balls white, 4 black.
6. 5 balls white, 5 black.
7. 4 balls white, 6 black.
8. 3 balls white, 7 black.
9. 2 balls white, 8 black.
10. 1 ball white, 9 black.

On the *first* hypothesis, the probability of the observed event is certain *i. e.* 1. On the *second* hypothesis, the probability is $\frac{9}{10}$. On the *third* hypothesis, $\frac{8}{10}$; on the *fourth* hypothesis, $\frac{7}{10}$; on the *fifth* hypothesis, $\frac{6}{10}$; and so on.

Now the probability of the fifth hypothesis being the true hypothesis is proportional to $\frac{6}{10} = k\frac{6}{10}$ (say). Since before the event, each was equally likely; but since one of the hypotheses must be the true hypothesis, $k\ (1 + \frac{9}{10} + \ldots\ldots\ldots\ldots + \frac{1}{10})$ must be 1; *i. e.* $k.\ \frac{55}{10} = 1$. $\therefore k = \frac{10}{55}$. Hence the probability of 6 balls being white is $k.\ \frac{6}{10}$ *i. e.* $\frac{10}{55} \times \frac{6}{10}$ *i. e.* $\frac{6}{55}$.

§ 7. **Probability and Induction** :—There are two views as to the relation between Probability and Induction.

(i) Probability is based on Induction.—Dr. Venn.

(ii) Induction is based on Probability.—Jevons.

If we take the senses in which the words—Induction and Probability have been used in Logic, we must acknowledge that the ground of Probability is *experience*,—that Induction, humanly speaking, does not rest on Probability, but that the probability of concrete events rests on Induction and therefore on Causation.

Compare the inferences :—

(i) All known A are B ;
$\therefore$ All A are B.

(ii) $\frac{5}{8}$ known A are B;

$\therefore$ $\frac{5}{8}$ of all A are B.

Both of these inferences are Inductions. So Probability is based on Induction.

(We take Inductive conclusions as certain, when they are in accordance with *our* standard of certainty. But Jevons does not agree with this view; he considers all Inductive conclusions as more or less *probable* only. According to Jevons, Inductive conclusions can never be considered as certain. The more is the number of cases examined,—the greater is the probability, so Jevons maintains that Induction is based on Probability.)

These two views, according to Dr. Keynes, **may be reconciled.** Dr. Venn's view is a simple one; he took the *statistical Probability* into consideration. Jevons had *Intellectual Probability* in his view.

§ 8. Probable and Proportional Propositions :— Every probable proposition is connected with what Dr. Venn calls a Proportional proposition of the form "5 A in 8 are B." It can be shown that every Paobable proposition must ultimately be traced to a Proportional proposition of that form, and that, without tracing it to such a proposition, we can give no rational account of its meaning, when the probability is represented by a fraction. From the Proportional proposition "5 A in 8 are B", we may infer that "Any A is *probably* B," the probability being represented by the fraction $\frac{5}{8}$. Conversely from a probable proposition we may infer a proportional one. Given the probable proposition "A is *probably* B",

the probability of which is represented by the fraction $\frac{2}{3}$, we may infer the proportional proposition "2 in 3 A are B."

§ 9. Some Calculations of Probabilities :—

(1) *When one of the events is certain and the other only possible or probable.* What will be the possibility of their occuring together ?

For example, in drawing lots or casting dice the event of drawing or casting ts certain, but that of getting the lucky lot or die is only possible.

If it be 1 in 6 or 10 or 100, then the probability will be $\frac{1}{6}$ or $\frac{1}{10}$ or $\frac{1}{100}$.

(2) *If two events are independent, having neither connexion nor repugnance,* the probability of their concurring is found by multiplying together the separate probabilities of each occurring. In such cases we have to estimate first the probability of each separately, after which the probability of their concurrence will be obtained by multiplying their separate probabilities together.

Suppose the question of the concurrence of a dry day with a westerly wind. Suppose we have found that there are generally 2 dry days to 3 rainy ones, giving a probability $\frac{2}{3}$; and that one day in these days has westerly wind, giving the probability of $\frac{1}{3}$. Then the probability of a dry day with a westerly wind will be $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ *i. e.* we may expect two such days, for seven different ones.

(3) *If two events or causes do not concur, the probability of one or the other occurring is the sum of the separate possibilities.* If the chances of two exclusive or incompatible events be

respectively $\frac{1}{X}$ and $\frac{1}{Y}$, the chance of one or other of their happening will be $\frac{1}{X} + \frac{1}{Y}$.

For example; if 1 in 1000 is burned and 2 in 1000 are drowned, the probability of being burnt or drowned is $\frac{1}{1000}$ +

XI

THE GROUND OF INDUCTION.

§ 1. **The meaning of "the Ground of Induction":**—In Inductive reasoning we come to a general conclusion after observing something to be true, in some particular cases. How are we justified in doing this? An answer to this question is the statement of the ground of Induction. The Ground of Induction is the reason we can state for our generalization from experiences of some particular facts.

§ 2. **The assumption involved in Inductive Reasoning: Uniformity of Nature.**—In every case of Induction there is an assumption involved with regard to the course of nature and the order of the universe; namely that there are such things in nature as parallel cases; that what happens once will under a sufficient degree of similarity of circumstances, happen again, and not only again, but as often as the same circumstances recur. This means that the course of nature is uniform,—that the universe is governed by general laws. This is the fundamental principle or the general axiom of Induction.

§ 3. **The Uniformity of Nature: The Grounds of Induction.**—The Uniformity of Nature is the *Objective ground* of Induction. Wherever uniformity exists, there we can draw inferences. This uniformity is our warrant for all inferences from experience. If nature had not been uniform,

we could not have drawn any conclusion regarding the unknown from the observation of the known.

Again we can not reason inductively unless we possess (*a*) the power of observation, (*b*) the faculty of memory and (*c*) the belief in the uniformity of Nature as distinct from, but corresponding to the objective uniformity itself. So these principal postulates demanded on the mental side constitute what is called the *Subjective ground* of Induction.

When we are asked to assign the *Material ground* of Induction, we can say that as we cannot reason inductively without observing particular facts and the operation of the Law of causation in them, Observation and Experiment (which is also Observation under prepared and therefore known conditions) may be regarded as such.

The Uniformity of Nature is observed principally as,—(*a*) Uniformity of Co-existence, and (*b*) Uniformity of Succession or which is the result of the Law of Causation. In the actual working of Induction, we find it to be chiefly concerned with the second kind of uniformity. The principle of causation which produces uniformity of nature is regarded as the *Formal ground* of Induction; and the Inductive canons derived from it are means of testing the *formal* sufficiency of observations to justify the statement of a law. If we can observe the process of *cause* and *effect* in nature, we may generalize our observation into a law, because that process is invariable.

§ 4. What is meant by the Uniformity of Nature?

When we say that the "Uniformity of Nature" is the basis of our Inductive inference, "uniformity" then is not to be

taken to mean "resemblance." It is identity alone, not in mere resemblance, that we can find a firm basis of inference. By the "uniformity of nature" then we do not mean, that "the unknown will be similar to the known, that the future will resemble the past." Such a general assumption that the future will resemble the past we have no right to make. From mere likeness of isolated phenomena we can draw no safe conclusions at any time, but only from "identity of conditions." What is true at one time under any set of conditions, is always true under that set of conditions. So the uniformity of nature implies the Universality of Law. The Universality of a Law means that phenomena are subject to a certain law which we can discover.

§ 5. How do we know that Nature is uniform? Our conception of the Unity of Nature.—According to Green the fundamantal idea which underlies all our attempts to attain knowledge is that of the Unity of Nature,—of the world as a system. If there had not been universal laws there would have been no Unity of Nature. By "Unity of Nature" we do not mean that the universe is an unchanging identity, but that it is a system which remains identical with itself amidst the unceasing changes of relations between its parts. But every relation and consequently every change of relation is universal; *i. e.*, it holds true everywhere and always, of all identical facts. Hence the idea of unity explains that of uniformity. Unity implies uniformity.

Uniformity is a form without which intelligible experience would be impossible. *It cannot be derived from mere repeated sensuous experience*, as Mill and other empiricist writers hold.

Such a derivation would be possible only, if observation gave us nothing but instances of uniformity. The chaos which nature really presents on the first observation really vanishes upon deeper investigation. But why do we investigate it at all? Because we believe in the Uniformity of Nature. How can this be, if the supposition is only derived from the observation of uniformity—an observation which presupposes the interrogation. It is evident then that the principle of uniformity is not derived from sensuous experience.

Nor it is an innate principle born ready-made with every man. Mill has proved it very clearly. He points out that, as a general maxim it has scarcely entered into the minds of any but philosophers. But this fact is also fatal to Mill's own position, that the principle is derived from observation. Sigwart justly says:—" If we had needed merely to open our eyes in order to see uniformity in the course of nature everywhere before us, belief in the thoroughgoing constancy of the way in which causes act, would not have been so slow to arise, nor have been still only a scientific and not a popular belief, nor would the tendency to make capricious powers, demons and gods responsible for what happens in the universe have been so deeply rooted."

Our belief in the Uniformity of Nature really arises from our conception of Unity of Nature.

§ 6. Mill's View as to our knowledge of the Uniformity of nature:—How do we know that Nature is uniform? According to Mill we know it by Inductive reasoning. We observe uniformity in a particular department of the universe,—then in another department,—

then in a third department,—then in many other departments, and at last we come to the Inductive conclusion that Nature is uniform. Suppose we knew nothing of the principle *i. e.*, did not know whether Nature was uniform or not, yet when we inferred that "All men are mortal," after observing A, B and C's death, we could conclude that Nature was uniform in respect, at least, of the connexion between men and mortality; so also in a second Induction we might prove Nature to be uniform in another respect; in a third, and so on. As fresh instances of proved uniformity were added to our list, we might begin to suspect that Nature was always uniform; new cases of Induction constantly being made, each in its own sphere proving Nature uniform, would proportionally strengthen that suspicion; and when finally age after age passes away and Inductions innumerable are made, every one of which adds its item of proof without a single contradictory instance, the inference of the universality of the principle is irresistible.

Criticism of Mill's View:—Mill condems Induction by Simple Enumeration; but his knowledge of the Uniformity of Nature which is the ground of Induction is itself based on Induction by Simple Enumeration. Mill says:—"Far from being the first Induction, we make, it is one of the last or at all events one of those which are latest in attaining strict philosophical accuracy." So this derivation of the fundamental principle or general axiom of Induction from "facts observed" renders the whole process of Induction nugatory as a means of attaining knowledge.

§ 7. According to Mill Induction by Simple

Enumeration may sometimes be valid:—Mill says, —This method is valid precisely in proportion to our assurance that if an exception ever did occur, we should know of it; in other words precisely as the subjects-matter is limited and special, so is the process insufficient and delusive. As its sphere widens, this unscientific method becomes less and less liable to mislead. The axiom of the Uniformity of Nature is proved by this form of Induction; the evidence consisting in this, that the principle has been found true in every legitimate Induction hitherto made and never once false; while at the same time, from the fact that these innumerable Inductions cover the whole field of Nature's operations, we are entitled to conclude that any real exception must have come under our notice.

§ 8. The sense in which Mill regards the Uniformity of Nature as the ground of Induction:—An Induction may be thrown into the form of a syllogism by supplying a major premiss, thus:—

Whatever is true of A, B, C is true of all men;
A, B, C are mortal;
∴ All men are mortal.

Now it is evident that this major premiss is nothing else but an assertion of the uniformity of Nature in so far as regards the phenomena we are concerned with. "Uniformity in Nature" means that what we find in one or more cases we shall continue to find in similar cases, which is exactly our major premiss.

[According to Mill, the major premiss of every syllogism is no part of the evidence which proves the conclusion, but only a

mark that there is sufficient evidence to prove the conclusion. Mill maintains that both the conclusion and the major premiss are alike conclusions from the antecedent observed particular cases.] We see, A, B, C and everybody else whom we have observed, die;—this is our evidence; and from it we conclude "All men are mortal;" but it is that very evidence that gives us our assurance that "what in this respect is true of A, B, C, is true of all men," in other words our observed cases prove to us that Nature is uniform in respect of the connection between humanity and mortality. [The Uniformity of Nature stands in the same relation with all Inductive conclusions as the major premiss of every syllogism stands in relation to its conclusion.] A syllogistic conclusion, according to Mill, is regarded as reliable when it is shown to be consistent with the major premiss, it being a wider generalisation. The Uniformity of Nature and an inductive inference are both conclusions drawn from the observation of particular facts; so the Uniformity of Nature which is a wider generalisation only, confirms other inductive conclusions.

§ 9. **The Uniformity of Nature and Laws of Nature**:—According to Mill, the general regularity which we find in Nature, results from the co-existence of partial regularities. (The Uniformity of Nature is a complex fact, compounded of all the separate uniformities which exist in respect to single phenomena.) These various uniformities, when ascertained by a sufficient Induction are ordinarily called Laws of Nature. (Strictly speaking a law of nature is any established uniformity which cannot be accounted for by or resolved into simpler uniformities.)

Symbolic Example :—If A is always accompanied by D, B by E and C by F, it follows, that AB is accompanied by DE, AC by DF, BC by EF and finally ABC by DEF; here we have, ordinarily speaking, seven laws of nature; and strictly speaking only the first three laws, if they are not resolvable into any other simpler laws, are called Laws of Nature; and others are called simply laws.

§ 10. The Ground of Induction and Laws of Nature :—All Laws of Nature are inductive conclusions. But experience testifies that among the uniformities which it exhibits, some are more to be relied on than others. These stronger Inductions are the test to which we endeavour to bring the weaker ones. Suppose, for instance, that we possess a strong induction to this effect,—"every effect must have a cause," it is evident that if by any means we can bring a weaker generalisation within this better established law *i. e.* if we can show that either the weaker generalisation must be true or our strong induction must be false,—the weaker is at once raised to the same degree of certainty with the stronger. This mode of correcting one generalisaton by means of another, a narrower generalisation by a wider, which common sense suggests and adopts in practice is the real type of scientific Induction. In this sense, Mill considers the Uniformity of Nature or the Law of Causation which is its principal aspect as the ground of all Induction.

§ 11. The Uniformity of Nature and the Law of Causation :—For the theory of Induction, the specially important aspect of the Uniformity of Nature is the Law of Causation.

§ 12. The statement of the Law of Causation :- Every phenomenon which has a beginning must have a cause; and it will invariably arise whenever that certain combination of positive facts which constitutes the cause exists, provided certain other positive facts do not exist also.

This law contains two clauses :—

(1) That every phenomenon which has a beginning must have *some* cause.

(2) Given the cause, the effect will invariably follow, provided that counteracting causes do not exist.

Remarks :—It is to be noticed that a phenomenon or a set of phenomena is considered here as the cause of another phenomenon. In Logic we are concerned with phenomenal cause only.

§ 13. Aristotle's Account of Causation :—The Aristotelian account is classical. He distinguishes four kinds of causes requisite for the existence of a thing :—

(i) **Final cause** *i.e.*, the end or motive or purpose determining the production of a thing.

(ii) **Material Cause** *i.e.*, the matter or substance which composes it.

(iii) **Efficient Cause** *i.e.*, the active force employed in its production.

(iv) **Formal Cause** *i.e.*, the pattern, idea or essence in accordace with which it is produced.

Example :—What is the cause of a table? The cause is a complex one, involving the following factors :—Final cause :— To rest book; Material Cause :—Wood; Efficient Cause :—

Work of the carpenter or machine ; Formal cause :—Idea or design of the carpenter.

Remarks :—In Logic, the discussion of such causes is unnecessary. The Final Cause theory brings Theology. If by "matter" we are to understand logical subject-matter, it is always present and does not help us in drawing up rules of inference. Efficient causes are often spoken of as if they were a distinct *kind* of cause, but what is meant is that efficiency is an element invariably present in every case. as something to be added on to the mere regularity inorder to complete the conception of cause. But its introduction here adds no further element of information, so far as inference is concerned.

The theory of Formal Cause is also unnecessary in Logic. Given any particular example of causation, say the melting of wax by fire, to determine exactly what is here meant by the *form* and the *matter* respectively seems a rather helpless piece of subtlety.

Mill says :—"The only notion of a cause which the theory of Induction requires, is such a notion as can be gained from experience. The Law of Causation, the recognition of which is the main pillar of inductive science, is but the familiar truth, that invariability of succession is found by observation to obtain between every fact in nature and some other fact which has preceded it ; independently of all considerations respecting the ultimate mode of production of phenomena and of every other question regarding the nature of things-in-themselves."

§ 14. Dr. Martineau on "Cause" :—According to Dr. Martineau, "a phenomenon cannot be the cause of another phenomenon. The idea of the productive power is

involved in the idea of cause. So some power or force existing along with phenomenon is regarded as cause. But no effect can be produced unless that force is excercised in a certain definite manner *i.e.*, directed to a certain definite channel. A force can be so directed only by an intelligent person. The intelligent person, by which the forces operating in the universe have been directed is the only cause of the universe which is the result of such direction; God is that intelligent person and He is the cause of everything.

Remarks :—This theory of cause, though a correct one, is not to be accepted for the purpose of Inductive Logic. In Inductive Logic we have to establish universal relation between facts or phenomena; so the theory of phenomenal cause is to serve our purpose.

§ 15. Hume's Definition of Cause :—The cause of any fact is the antecedent fact which it invariably follows.

Reid's Criticism :—Invariability of succession cannot establish the relation of cause and effect between any two facts or phenomena. According to Hume's definition of cause, we have to regard night as the cause of day, and day as the cause of night; but we know night is not the cause of day and day is not the cause of night.

§ 16. Mill's Definition of Cause :—The cause of any fact or phenomenon is that antecedent which it invariably and unconditionally follows.

Mill says :—When we define the cause of anything to be "the antecedent which it invariably follows," we do not use this phrase as exactly synonymous with "the antecedent which it invariably *has* followed in our past experience." Such a

mode of conceiving causation would be liable to the objection very plausibly urged by Dr. Reid, namely that according to this doctrine night might be the cause of day, and day the cause of night; since these phenomena have invariably succeeded one another from the begining of the world. But it is necessary to our using the word cause that we should believe not only that the antecedent always *has* been followed by the consequent, but that it always *will* be so. And this would not be true of day and night. We do not believe that night will be followed by day under all imaginable circumstances, but only that it will be so, provided the sun rises above the horizon,—there is the rotation of the earth round the Sun. So the cause of any phenomenon is not that which it *invariably* follows but that which it *invariably* and *unconditionally* follows. This is what writers mean when they say that the notion of cause involves the idea of *necessity*. If there be any meaning which confessedly belongs to the term **necessity**, it is **unconditionalness.** Invariable sequence, therefore, is not synonymous with causation, unless the sequence besides being invariable is unconditional.

[“The *unconditional* invariable antecedent is termed the cause] That which would not be followed by the effect unless something else had preceded, and which if that something else had preceded, would not have been required, is not the cause, however invariable the sequence in fact be.”—*Mill.*

§ 17. The three different views regarding the phenomenal cause and the phenomenal effect :—

(i) The Popular View.

(ii) The Scientific View or the Brown-Herschel-Mill View.

(iii) The Strictly Philosophical View.

(i) **The Popular View** :—The characteristic of the Popular View is that it singles out *one* antecedent circumstance or phenomenon and *one* such consequent and regards them as cause and effect.

Example : – People say that the spark of fire is the cause of the explosion ; – the passage of the bullet through the body is the cause of one's death. In the case of the explosion we must take into account the proxemity of the powder, its dryness, the presence of oxygen in the atmosphere &c. &c. ; in the other case, the speed of the bullet, the part of the body pierced, the general state of the man's health.

Criticism :—The cause of an event is not generally one single phenomenon ; but, the effect is produced really by a combination of phenomena. So it is inaccurate to mention any single phenomenon as the cause.

(ii) **The Scientific View or the Brown-Herschel-Mill View** : – According to this view the cause of any event is not one single antecedent circumstance or phenomenon, but *an assemblage of phenomena*, which occurring, that event follows invariably and unconditionally. The effect is one single phenomenon ; but the cause is an assemblage of phenomena, because the effect really takes place owing to the presence of all the antecedent phenomena. Here although we say that our cause includes all the antecedents, but we really omit a quantity of determining elements solely on the ground of their comparative insignificance. According to this view an effect may be produced by many causes.

Criticism : – As the whole cause is made up of several

phenomena, so the whole effect is also made up of several phenomena. But this view singles out one phenomenon only as "effect." We have to find out generally the cause of one such single phenomenon; so this view is useful for practical purposes.

But this view simply *enunciates* the *law* of causation—that the cause A is invariably followed by the effect *a*. But it does not *explain* **why** it is so; it merely says *that it is so* So that it does not explain the principle of causality at all. This fact of *invariable succession* has got to be *explained*. Thus this view, however important from the practical point of view, is superficial and does not touch the real question at all.

(iii) **The Strictly Philosophical View**:—According to this view both the cause and the effect are made up of several phenomena;—all the antecedent phenomena make up the cause and all the consequent phenomena make up the effect. This view does not admit the Plurality of Causes. An effect is always to be produced, according to this Philosophical view, by the same cause.

Criticism:—If we take this view of cause, we do not find in nature the same cause occurring again; so the same effect is never repeated. We want to know the cause, so that we may produce the effect; or when the effect is produced we may assign its cause. Consequently this view is not useful for practical purposes.

§ 18. Popular distinction between the "conditions" and the "cause" of an effect is unphilosophical.

It is seldom, if ever, between a consequent and a single

antecedent, that the causal relation subsists. It is usually between a consequent and the sum of several antecedents; the occurrence of all of them being requisite to produce, that is, to be certain of being followed by, the consequent. In such cases it is very common to single out one only of the antecedents under the denomination of Cause, calling the others merely Conditions.

Thus, if a person eats of a particular dish, and dies in consequence, that is, would not have died if he had not eaten of it, people would be apt to say that eating of that dish was the cause of death. There need not, however, be any invariable connexion between eating of the dish and death; but there certainly is, among the circumstances which took place, some combination or other on which death is invariably consequent; as for instance, the act of eating of the dish, combined with a particular bodily constitution, a particular state of present health, and perhaps even a certain state of the atmosphere: the whole of which circumstances perhaps constituted in this particular case the set of antecedents which determined it, and but for which it would not have happened. The real cause is the whole of these antecedents; and we have, philosophically speaking, no right to give the name of cause to one of them exclusively of the others.

All the circumstances are equally indispensable to the production of the consequent, and the statement of the cause is incomplete, unless in some shape or other we introduce them all. A man takes mercury, goes out of doors, and catches cold. We say, perhaps, that the cause of his taking cold was exposure to the air. It is clear, however, that his having taken

mercury may have been a necessary circumstance of his catching cold; and though it might be consistent with usage to say that the cause of his attack was exposure to the air, to be accurate we ought to say that the cause was exposure to the air while under the effect of mercury.—Mill.

§ 14. Distinction between the factors of the cause as "agent" and "patient" is merely verbal.

One of the positive factors composing the cause is sometimes called an *agent*, and another as a *patient*. Both of these, it would be universally allowed, are the factors of the cause. But the one which is regarded as acting is called the *agent* and the other which is regarded as acted upon is called the *patient*. In such a case the name "cause" is generally given to the factor which is called the *agent*.

"Those who have contended for a radical distinction between *agent* and *patient*, have generally conceived the agent as that which causes some state of, or some change in the state of another object which is called the patient. But *states* of objects are also active phenomena." If a man be poisoned by prussic acid, the poison would be reckoned as an "*agent*," the nervous system of the individual as the "*patient*" though prussic acid is called the agent of a person's death, the whole of the vital and organic properties of the patient are as actively instrumental as the poison, in the chain of effects which so rapidly terminates his sentient existence.

In the process of education one may call the teacher the agent, and the scholar only the material acted upon; yet in truth all the facts which pre-existed in the scholar's mind exert

either co-operating or counteracting agencies in relation to the teacher's effort.

It is not light alone which is the agent in vision, but light coupled with the active properties of the eye and brain, and with those of the visible object.

So the distinction between agent and patient is merely, verbal, patients are also always agents, and Cause includes both the *agent* and the *patient*.

§ 18. What phenomenon is generally selected as the cause out of the antecedent phenomena of the event :—

(i.) That one of the antecedents which comes last, and is thus *an event* completing the sum of conditions, which forms the cause, and upon which the effect immediately follows, is termed *the* cause.

(ii.) That one of the antecedents which is most peculiar and special to the aggregate of antecedents, is often popularly *the* cause.

(iii.) So also that one of the antecedents which is least likely to be known to the hearer.

Mill says :—" Thus we see that each and every condition of the phenomenon may be taken in its turn, and with equal propriety in common parlance, but with equal impropriety in scientific discourse, may be spoken of as if it were the entire cause."

§ 19. Why all the conditions of the cause are not enumerated even when we aim at accuracy :—

(i.) Because some of them will in most cases be understood without being expressed.

(ii.) Because for the purpose in view they may without detriment be overlooked.

For example, when we say, the cause of a man's death was that his foot slipped in climbing a ladder, we omit as a thing unnecessary to be stated the circumstance of his weight, though quite indispensable a condition of the effect which took place.

When we say that the assent of the crown to a bill makes a law, we mean the assent, being never given until all the other conditions are fulfilled, makes up the sum of the conditions, though no one now regards it as the principal one.

§ 22. **Positive and Negative conditions forming the whole cause:**—The *positive conditions* are those circumstances, owing to whose presence the effect occurs. The negative conditions of an effect may be summed up in this:—absence of *counteracting* or *preventing* causes.

(*a*) *Counteracting causes.*—Most causes counteract the effects of other causes by the operation of the very same law as that by which they produce their own effects.

(*b*) *Preventive causes:*—Some causes seem to be simply preventive *i.e.*, destroying an effect, not by producing their own, but by simply arresting it.

23 § **Is the cause of an event to be regarded as its immediate antecedent?**—"That the cause of any event is an *immediate* antecedent follows from its being an *unconditional* one. For if there are three events A, B, C, causally connected, it is plain that A is not the unconditional antecedent

of C, but requires the further condition of first giving rise to B. But that is not all; for the B that gives rise to C is never merely the effect of A; it involves something further."—Carveth Read.

"The object of securing this close approximation between the elements of the sequence is obvious. It is done simply in order to secure regularity. A remote sequence can never be a certain one."—Dr. Venn.

§ 24. The Fallacy of arguing "Post hoc, ergo propter hoc," or "Cum hoc, ergo propter hoc :—

The cause is the *invariable* and *unconditional* antecedent of the effect. Accordingly, not every antecedent of an event is its cause: to assume that it is so, is the fallacy of arguing "post hoc ergo propter hoc."

XII

EXPERIMENTAL METHODS.

§ 1. The Inductive or Experimental Methods; their Utility :—There are some methods with the help of which we can find out the cause of a given phenomenon or the effect of a given cause. When the causal relation is established between the anticedent and the consequent phenomena, we can come to a universal truth regarding them *i. e.*, we can get an Inductive conclusion established. These methods are called the Inductive Methods or the Experimental Methods.

§ 2. What are the Inductive Methods ?—The Inductive Methods are the Methods of Observation and Experiment. The following are the Inductive Methods or the Methods of Experimental Enquiry :—

(i) The Method of Agreement.
(ii) The Method of Difference.
(iii) The Joint Method of Agreement and Difference.
(iv) The Method of Residues.
(v) The Method of Concomitant Variations.

§ 3. The Canons of the Methods and their Symbolic Formulæ :—

(i) Method of Agreement :—

"If two or more instances of the phenomenon under investigation have only one circumstance in common, the

circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon."

Symbolic Formula :—

ABC—*abc*
ADE—*ade*
AFG—*afg*
∴ A is the cause of *a*.

(ii) Method of Difference :—

" If an instance in which the phenomenon under investigaton occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occuring only in the former ; the circumstance in which alone the two instances differ, is the effect or the cause or indispensable part of the cause of the phenomenon.

*Symbolic Formula :—*ABC—*abc*
BC—*bc*
∴ A is the cause of *a*.

(iii) Method of Agreement and Difference :—

" If two or more instances in which the phenomenon occurs, have only one circumstance in common, while two or more instances in which it does not occur, have nothing in common save the absence of that circumstance ; the circumstance in which alone the two sets of instances differ, is the effect or the cause or an indispensable part of the cause of the phenomenon."

Symbolic Formula :—

$$\left.\begin{array}{l} \text{ABC—}abc \\ \text{ADE—}ade \\ \text{AFG—}afg \end{array}\right\}$$

$$\left.\begin{array}{l}\mathrm{BDM}—bdm\\ \mathrm{CEO}—ceo\\ \mathrm{FGK}—fgk\end{array}\right\}$$

∴ A is the cause of *a*.

(iv) Method of Residues :—

"Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents and the residue of the phenomenon is the effect of the remaining antecedents."

Symbolic Formula :—B and C are known to be the causes respectively of *b* and *c*.

ABC—*abc*

BC—*bc*

∴ A is the cause of *a*.

(v) Method of Concomitant Variations :—

"Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon or is connected with it through some fact of causation."

Symbolic Formula :—

$A_1BC—a_1bc$

$A_2BC—a_2bc$

$A_3BC—a_3bc$

∴ A is the cause of *a*.

§ 4. Inductive Methods and the Law of Causation :—

The validity of all the Inductive Methods depends on the assumption that every event or the beginning of every phenomenon must have some cause, some antecedent, on the exis-

tence of which it is invariably and unconditionally consequent. These methods are used by Mill, in strict sub-ordination to the Law of Causation.

In the Method of Agreement this is obvious; that method avowedly proceeding on the assumption that we have found the true cause as soon as we have negatived every other. This assertion is equally true of the Method of Difference. That method authorizes us to infer a general law from two instances; one in which A exists together with a multitude of other circumstances, and *a* follows; another in which A being removed, and all the circumstances remaining the same, *a* is prevented. What does this prove? It proves that *a* in the particular instance cannot have had any other cause than A; but to conclude from this that A was the cause or that A will on other occasions be followed by *a*, is only allowable on the assumption that *a* must have some cause; that among its antecedents in any single instance in which it occurs, there must be one which has the capacity of producing it at other times. The same thing is true of the other Inductive Methods.

§ 5. The General Principle involved in the Inductive Methods:—The general principle involved in these methods is that of *Elimination.* This was explained by Mill to mean the successive exclusion of the various circumstances which are found to accompany a phenomenon in a given instance, in order to ascertain what are those among them which can be absent consistently with the existence of the phenomenon. What cannot be removed without removing the given phenomenon is necessary to the production of the phenomenon *i. e.*, is part of its cause.

The Method of Agreement stands on the ground that whatever can be eliminated is not connected with the phenomenon by any law. The Method of Difference, on the contrary, rests on the foundation that whatever cannot be eliminated is connected by a law with the phenomenon. If these methods, especially that of Difference, can be strictly applied, there is no doubt that their results will be universally true, and such results have been obtained in science by employing the Method of Difference. But there is always a possibility of eliminating or of introducing more than one element at a time, and this causes an uncertainty as to the connexion of the phenomenon with its supposed cause.

We must remember that the Method of Residues and the Method of Concomitant Variations are only special applications of the Method of Difference.

§ 6. Every one of these Methods has its special utility.

The Methods of Agreement and Difference:—The Method of Agreement leads only to laws of phenomena—to uniformities, which either are not laws of causation, or in which the question of causation must for the present remain undecided. But the Method of Agreement is chiefly resorted to (i) as a means of *suggesting* applications of the Method of Difference; or (ii) as an inferior resource, in case the Method of Difference is impracticable, which impracticability generally arises from the impossibility of artificially producing the phenomena. The Method of Agreement is more especially the resource employed where experimentation is impossible. The method of Difference is more particularly

a method of artificial experiment. The Method of Difference generally affords a more efficacious process, which can ascertain causes as well as mere laws. If the instances fulfil exactly the requirements of the canon, this method is perfectly rigorous in its proof.

The Joint Method of Agreement and Difference :—In cases, in which it is not possible to obtain the precise pair of instances which the canon of the Method of Difference requires—instances agreeing in every antecedent except A or in every consequent except *a*, we may be able by a double employment of the Method of Agreement, to discover in what the instances which contain A or *a* differ from those which do not. So the Joint Method of Agreement and Difference is useful.

Suppose we examine a variety of instances in which *a* occurs, and find them agree in containing A; so we again observe a variety of instances in which *a* does not occur, and find them agree in not containing A, but containing the other factors found in the previous sets of instances, which establishes by the Method of Agreement, the same connexion between the absence of A and the absence of *a*, which was before established between their presence. This method is called the Indirect Method of Difference or the Joint Method of Agreement and Difference; and consists in a double employment of the Method of Agreement, each proof being independent of the other and corroborating it. It can be regarded as a great extension and improvement of the Method of Agreement. It is called the Indirect **Method of Difference**, because like the Method of Difference, it proceeds by ascertaining how

and in what, the cases where the phenomenon is present, differ from those in which it is absent. It is called the **Indirect** Method of Difference, because the negative instance is not obtained by *direct* experiment, but *indirectly*, by showing what would be the result if experiment could be made.

The Method of Residues:—Of all the methods of investigating laws of Nature, the Method of Residues is the most fertile in unexpected results; often informing us of sequences in which neither the cause nor the effect was sufficiently conspicuous to attract of themselves the attention of observers. The agent A may be an obscure circumstance, not likely to have been perceived unless sought for until attention had been awakened by the insufficiency of the obvious causes to account for the whole of the effect. And *a* may be so disguised by its intermixture with *b* and *c*, that it would scarcely have presented itself spontaneously as a subject of separate study.

The Method of Concomitant Variations:—There remains a class of laws which can be ascertained by the Method of Concomitant Variations; namely the laws of those permanent causes or indestructible natural agents, which it is impossible either to exclude or to isolate; which we can neither hinder from being present, nor contrive that they shall be present alone. But though we cannot exclude an antecedent altogether, we may be able to produce or nature may produce for us, some modification in it. By a modification is here meant a change in it, not amounting to its total removal. This Method is like the Method of Residues a special application of the Method of Difference.

If some modification in the antecedent A is always followed by a change in the consequent *a*, the other consequents *b* and *c* remaining the same; or *vice-versa*, if every change in *a* is found to have been preceded by some modification in A, none being observable in any of the other antecedents; we may safely conclude that *a* is wholly or in part, an effect traceable to A, or at least in some way connected with it through causation.

For example, in the case of heat, though we cannot expel it altogether from anybody we can modify it in quantity, we can increase or diminish it; and doing so, we find that such increase or diminution of heat is followed by expansion or contraction of the body. In this manner we arrive at the conclusion, otherwise unattainable by us, that one of the effects of heat is to enlarge the dimensions of bodies.

§ 7. The Method of Agreement and the Method of Difference contrasted:—

(1) The Method of Agreement stands on the ground that whatever can be eliminated, is not connected with the phenomenon by any law. The method of Difference has for its foundation that whatever cannot be eliminated, is connected with the phenomenon by a law.

(2) In the Method of Difference, the instances agree in everything except in the possession of two circumstances which are present in the one instance and absent in the other. In the Method of Agreement, the various instances compared (for here we generally require more than two instances) agree in nothing, except in the possession of two circumstances, which are comman to all the instances.

(3) One method is called the Method of Agreement, because we compare *various instances* to see in what they *agree*; the other is called the Method of Difference, because we compare *an instance* in which the phenomenon occurs with *another* in which it does not occur, in order to see in what they *differ*.

(4) The Method of Difference is specially adapted to the discovery of the effects of given causes, whereas if it is our object to discover the cause of a given effect, we are usually compelled to have recourse to the Method of Agreement.

(5) The Method of Agreement is, in fact mainly a Method of Observation; whereas the Method of Difference is mainly a Method of Experiment. The Method of Agreement is the resourse where experimentation is impossible.

§ 8. The Method of Difference and the Joint Method of Agreement and Difference :—The method of Difference compares two instances; the Joint Method compares two sets of instances. The proof derived from one set is independent of that derived from the other and corroborative of it. Still both together do not amount to a proof by the direct Method of Difference, on account of the possibility of the presence or of the absence of unknown antecedents in the possitive and negative sets respectively.

§ 9. The Method of Difference and the Method of Residues :—The Method of Residues is a modification of the Method of Difference; but the negative instance (*i. e.*, where phenomenon is absent) is obtained by Deduction, not by direct experience. The Deduction being this,—from the known effects of B and C separately we *infer* their effect conjointly, and subtract this effect from the total effect *abc*.

This method would be equally rigorous with the Method of Difference, if we could be certain (i) of the total effects of the known antecedents (B and C) and (ii) that the remaining antecedent (A) is the only one present.

§ 10. **Mill's Claims regarding the Inductive Methods** :—For these methods Mill makes very high claims. They are the only possible modes of experimental enquiry of direct induction *a posteriori*, as distinguished from Deduction. "Induction" says Mill, "is proof; it is inferring something unobserved from something observed; it requires, therefore, an appropriate test of proof; and to provide that test, is the special purpose of Inductive Logic." The business of Inductive Logic is to provide rules and methods to which if inductive arguments confirm, these arguments are conclusive and not otherwise. This is what the Inductive Methods profess to be.

§ 11. **Criticism of Mill's claims regarding the Methods** :—These claims are by no means universally granted by logicians. The balance of authority among modern logicians is against the claims of Inductive proofs and is not on their side. Mill himself also does not consistently maintain these high claims and also abundantly shows that the canons both demand the unattainable and fail to give conclusive proofs of the general propositions, which he held to be the business of Induction to discover and prove.

§ 12. **Criticism of the Method of Agreement** :— The Method of Agreement stands as one of these methods of "proof"—and sets out with claiming to establish both the effect of a given cause and the cause of a given effect.

But immediately after we are told, the conclusion by this method remains subject to very considerable doubt. This uncertainty arises from the impossibility of assuring ourselves that A is the *only* immediate antecedent common to all the instances. In other words it is impossible to fulfil the requirements of the canon.

But further, Plurality of Causes is a characteristic imperfection of the Method of Agreement which is essentially a method of Observation as distinguished from Experiment. Once grant the Plurality of Causes and the method ceases to furnish any valid proof. Mill himself says:—"If there are but two instances ABC and ADE, though these instances have no antecedent in common except A, yet as the effect may possibly have been produced in the two cases by different causes, the result is at most only a slight probability in favour of A." Hence the First canon is shown by its author to be false as the statement of a method of "proof." According to Mill, this method can only suggest the cause.

But Mill goes even further in the work of destruction. He tells us to *multiply* and *vary* our instances "all agreeing in no other antecedent except A"—a requirement which he has told us it is impossible to fulfil—as the only means of increasing the probability of our conclusion. Thus the Method of Agreement is shewn to be of very little use.

§ 13. **Criticism of the Method of Difference** :—The Method of Difference is regarded by Mill as the most valuable of all the Inductive Methods; it is according to him essentially the method of Experiment and it is claimed that by its agency both cause and effect can be established.

The method treated fairly, will not by itself justify the universal proposition that if a man is shot through the heart, he dies. Of course in such cases we do not argue inductively at all, but deductively from our knowledge of physiology and of firearms. To really test the method, therefore, we must put ourselves in the position of one who has no such knowledge, and who has never seen firearms or wounds inflicted by them. A person in such a position would not be justified in drawing a universal conclusion. The shot has caused death in *this* case; but is it due to the fact that it passed through the heart or to some individual peculiarity of the man; and if the former, would death always attend such a wound? The method gives no means of answering these questions.

Whilst affirming that plurality of causes does not affect the Method of Difference, Mill says,—"For if we have two instances ABC and BC, of which BC gives *bc*, and A being added converts it into *abc*, it is certain that *in this instance at least*, A was either the cause of *a* or an indispensable portion of its cause, even though the cause which produces it in other instances may be altogether different". But this is to acknowledge that plurality of causes does affect the method; and that consequently it can never prove a general proposition.

§ 14. **Mill's Symbolism Criticised**:—Mill's symbolism is defective. The use of A, *a*; B, *b*; C, *c*; &c. suggests that the causal connexions sought are already obtained; and this suggestion is strengthened by Mill's speaking of *a*, *b*, *c* as the consequents corresponding to the antecedents A, B, C. This would be an open begging of the question. Mill

probably does not mean this, but intends his symbols as mere empty forms. They are, however, undoubtedly confusing; if the correspondence between large and small letters mean nothing, it should not be employed; whilst if it has meaning, it begs the whole question. The methods might all be expressed by the formula 'AB—xy; AC—xz,' which avoids the objection just urged in Mill's symbolic statements.

Dr. Venn says:—"The common notation employing as it does the same letters, with the only distinction of making these capital or small inevitably suggests that there were a number of quite distinct cause-elements, each connected with a correspondingly distinct effect-elements, so that all which we had to do was to sort them out and assign each to its appropriate relative. I wish to keep clear of any suggestion of this kind and therefore write down the effect-elements, not as *b c d e* but *p q r s*; and thus avoid all suggestion that B, C, D produce *p*, *q*, *r* respectively, in the same way that A is considered to produce x."

§ 15. Are these Experimental Methods Inductive? By Induction, Mill continually tells us, he means inference from particulars. All processes of thoughts in which the ultimate premises are particulars are Induction. The Experimental Methods make their experiments directly upon complex cases. If we examine the canons of the Methods, we find they do not even profess to start from particular complex facts. They demand that the instances "have only one circumstance in common," "have every circumstance in common save one," "have nothing in common save the absence of that circumstance." Very little consideration is

needed to show that the complex particular facts of experience can never fulfil such conditions as these. Such facts never agree, still less do they differ in one point only. The cannons assume that many circumstances are first dismissed from consideration as "already known to be immaterial to the result." That is, they assume that the complex particulars of experience have already been analysed and that limited groups of antecedents consequents known to be causally connected have been separated out for the purpose of the inductive enquiry, whose task is only to obtain simpler causal connexion by eliminating some of the elements still left. The position from which we are invited to set out is very far from the beginning of experimental enquiry; the clear-cut instances supposed are possible only in an advanced stage of scientific research.

The Methods really start with universal judgments. The moment you have reduced your particular fact to a perfectly definite set of elements existing in relations which are accurately known, there you have left the fact behind you. You have already a judgment universal in the same sense in which the result of your induction is universal. The demand for a universal to start from is indeed made explicitly by the Fifth Canon which speaks of one phenomenon varying whenever another varies, thus postulating that a universal connexion between the two should be established before the method can be brought to bear upon it. Hence the methods which are held out to us as the only possible modes of experimental enquiry—of direct Induction *a posteriori*—are seen to presuppose the very work which they themselves are set forth as alone capable of accomplishing. It is evident then these

methods are not inductive in Mill's sense of the term, for they do not start from particular facts, but from propositions as universal as those they profess to prove.—Welton.

§ 16. **The Deductive reasoning is involved in these methods** :—If we examine the kind of reasoning which the methods involve, we find that it is deductive throughout. This is apparent at the first glance, in the Method of Residues; and is acknowledged by Mill: "Of the two instances which the Method of Difference requires, one positive, the other negative,—the negative one or that in which the given phenomenon is absent, is not the direct result of observation and experiment, but has been arrived at by deduction".

In so far as inference is involved in the use of any one of the methods, it is deductive in its essence. All the methods can be fairly represented by the formula AB—xy; AC—xz.

The argument then runs—

Any antecedent is invariably followed by the same consequent.

A is followed in this case by x and y (1st. example).

∴ A is invariably followed by x or y.

Taking this as the next major premiss, we have—

A is invariably followed by x or y.

But A is not invariably followed by y (2nd. example).

∴ A is invariably followed by x.

i. e., A is the cause of x.

By similar arguments it can be established from the premises that A is the cause x, and that B—y, C—z are also reciprocal causal connexions. The type of argument is seen to be deductive throughout, and to be valid if the truth and

sufficiency of the premises are granted; and if A, B, C and x, y, z symbolize elements which are independent of each other. But in no case is the inference "inductive," in the sense of drawing a conclusion more general than the premises.—Welton.

§ 17. **The sense in which the methods may be regarded as Inductive** :—It is true that the methods are not inductive in the empiricist sense, in which Mill uses the word, of enabling the enquirer to gather and prove universal laws by a simple comparison of particular facts: but they are inductive in the sense that their help is taken in inductive investigation.

§ 18. **The Inductive Methods of Agreement and Difference may be applied to fix the Definition of a word in ordinary use and in this application there is a postulate involved corresponding to the Law of Causation.**

The definition of a word should contain its exact connotation. By applying the Method of Agreement and that of Difference, we can determine that connotation. We know that whatever atribute can be absent without rendering the name inapplicable to the things which constitute its denotation is not a part of its connotation; and whatever attribute cannot be omitted without rendering the name inapplicable to the objects which it denotes, forms a part of its connotation.

When we observe the facts ABCD, ACDE, ABDF and ABEF whose common name is x, we find nothing but A in them common. This is the application of the Method of Agreement. It suggests that A is the connotation of x. Then

again by applying the Method of Difference, we find that A B C D is a thing which is called either Xy or X; and BCD is a thing which is y or not-X. Here we see that when A is absent, the thing is not called X; consequently there cannot be any doubt of A's being the connotation of X. The Method of Difference supplements the Method of Agreement. When the connotation is known in this way, the *definition* can be easily framed.

When we apply the Method of Agreement and that of Differnce in fixing *definition*, we take it for granted that the following postulate is given for carrying out the process :—

Every name is always to be used in the same sense *i. e.*, (i) every name has a fixed connotation and (ii) that is invariable.

This postulate corresponds to the Law of causation which says, (i) Every phenomenon has a cause and (ii) the same cause has the same effect; because it states (i) Every word has a fixed meaning and (ii) the same word must have the same meaning everywhere.

§ 19. The Method of Agreement and Empirical Laws :—We can never prove *causation* by the Method of Agreement; all that it proves that two phenomena (A and *a*) are found together,—an Empirical Law. We can never be sure that some unknown antecedent (B) is not either the cause of both phenomena, or causes one (*a*) while having been in our experience invariably conjoined with the other (A)

§ 20. The Method of Concomitant Variations and classification :—The application of the Method of concomitant Variations must be preceded by Classification by Series. The mere arrangement of a set of objects in a

series, according to the varying degree in which they exhibit some fact of which we are seeking the law, is naturally suggested by the necessities of our inductive operations.

§ 21. Difficulties in the application of the Experimental Methods :—

The great difficulty is the difficulty of discriminiating or distinguishing all the different antecedents and consequents and of eliminating precisely those antecedents which we wish to have eliminated and of retaining those which we wish to retain.

The difficulty of the investigation of the laws of phenomena by means of the Experimental Methods is singularly increased by the necessity of adverting to the following circumstances :—

(i) Plurality of Causes.

(ii) Intermixture of Effects.

(iii) Progressive Effects.

§ 22. Plurality of Causes :—By Plurality of Causes is meant that a given effect may arise from different causes in different cases.

Example :—*Death* may be caused by poison, disease, old age, gun-shot, violence, lightening &c.

§ 23. Plurality of Causes and the Method of Agreement :—The possibility that the same effect may have been produced by different causes leads to "the characteristic imperfection of the Method of Agreement. The Method of Agreement is vitiated by Plurality of causes. The effect *a* may be produced in one case by B ; in another by D ; in a third case by F. The Method of Agreement, therefore, cannot lead us to any certain conclusion. It can only suggest that

A may be the cause of *a*. So this method can be regarded as a Method of Suggestion only.

§ 24. Mill maintains that the imperfection of the Method of Agreement arising from the Plurality of Causes may be remedied.

(1) It is only when the instances, being indefinitely *multiplied* and *varied*, continue to suggest the same result, that this result acquires any high degree of independent value. When the instances are *varied and very many*, the supposition that the presence in all, of the common antecedent may be simply a coincidence, is rebutted; and this is the sole reason why mere number of instances differing only in immaterial points, is of any value.

(2) The uncertainty of the conclusion suggested by the Method of Agreement, may also be removed by having the conclusion confirmed by the application of the Method of Difference or by the Joint Method of Agreement and Difference.

§ 25. Plurality of Causes and the Method of Difference :—Mill maintains that the Method of Difference is not rendered imperfect by Plurality of Causes. For if we have two instances ABC and BC, of which BC gives *bc*, and A being added converts it into *abc*, it is certain that in this instance at least A was either the cause of *a*, or an indispensable portion of its cause. Plurality of Causes, therefore, not only does not diminish the reliance due to the Method of Difference, but does not even render a greater number of observations or experiments necessary; two instances, the one positive and the other negative, are still sufficient for the most complete and rigorous induction.

§ 26. Plurality of Causes and the Joint Method of Agreement and Difference:—The Joint Method of Agreement and Difference is not affected by the characteristic imperfection of the Method of Agreement. For in the Joint Method, it is supposed not only that the instances, in which *a* is, agree only in containing A, but also that the instances in which *a* is not, agree only in not containing A. Now if this be so, A must be not only the Cause of *a*, but the only possible cause: for if there were another, as for example B, then in the instances in which *a* is not, B must have been absent as well as A; and it would not be true that these instances agree *only* in not containing A. This constitutes an immense advantage of the Joint Method over the simple Method of Agreement.

The Method of Agreement, when applied to negative instances, is free from the characteristic imperfection which affects it in the affirmative cases. But it is generally altogether impossible to work the Method of Agreement by negative instances without positive ones. Hence the need of the Joint Method.

§ 27. How do we discover that a given effect is producible by a plurality of causes?

(1) When an effect is producible by two or more causes, they may be discovered as separate sequences by separate sets of instances.

One set of observations or experiments shows that the sun is a cause of heat, another that friction is a source of it, another that percussion, another that electricity, another that chemical action is such a source.

(2) Plurality may come to light in the course of collecting a number of instances, when we attempt to find some circumstance in which they all agree, and fail in doing so. We find it impossible to trace, in all the cases in which the effect is met with, any common circumstance.

§ 28. Intermixture of Effects:—Intermixture of effects takes place on account of a concurrence of two or more causes, not separately producing each its own effect, but interfering with or modifying the effects of one another. Of this there are two cases:—(i) Compound Effects and (ii) Heteropatic Effects.

Compound Effects:—In Compound Effects the separate effects of all the causes continue to be produced, but are compounded with one another, and disappear in one total. The effects of the different causes without being transformed altogether may be fused into one resultant in which they can no longer be distinguished and recognised.

Example:- In the principle of the Parallelogram of Forces, we have an instance of Compound Effect.

Heteropathic Effects:—We have an Heteropathic effect, when the joint effect is not of the same kind with the separate effects; the separate effects of the causes disappear and a totally new set is developed by their combination.

Example:—Chemical products.

§ 29. Progressive Effects:—A Progressive Effect is a complex effect arising from the operation of one cause, by the continual addition of an effect to itself.

Example:—The fall of heavy bodies to the earth, sixteen feet in the first second, forty-eight in the second and so on in

the ratio of odd numbers, from the action of one cause, gravity.

§ 30. **Temporary Effects, Permanent Effects and Progressive Effects** :—There is an obvious distinction between temporary and permanent effects. There are some phenomena, some bodily sensations for example, which are essentially instantaneous, and whose existence can only be prolonged by the prolongation of the existence of the cause by which they are produced. There are other phenomena permanent in their nature; having begun to exist, they would exist for ever unless some cause intervened having a tendency to alter or destroy them. Water once produced will not of itself relapse into a state of hydrogen and oxygen; such a change requires some agent having the power of decomposing the compound.

An agent or cause producing a permanent effect may instead of being merely temporary, be itself permanent. In this case whatever effect has been produced up to a given time would subsist permanently (absence of altering causes being supposed) even if the cause were then to perish. Since, however, the cause does not perish, being permanent, but continue to exist and operate, it continues to add more and more to the effect, and thus we get a *progressive effect* from the accumulating influence of a single permanent cause.

§ 31. **Progressive Effects are of two kinds** :—
(i) When the cause though constantly acting is not variable.
(ii) When the constantly-acting cause itself varies.

§ 32. **Complex Effects and Inductive Methods** :—

(i) In cases of **Compound Effects**, the Inductive

Methods are not applicapble. The Deductive Method is our only resource there.

(ii) There are some *Heteropathic Effects* which are called **Transformations**, where causes and effects are mutually convertible *i. e.*, when we can make A produce *a* or *a* produce A. Thus Hydrogen and Oxygen, when fused produce water; water galvanised produces hydrogen and oxygen. With the exception of "Transformations," the investigation of **Heteropathic Effects** by direct Induction is practised at such great disadvantages, as generally to be impracticable.

(iii) The questions, what effect will result from the continual addition of a given cause to itself and what amount of the cause being continually added to itself, will produce a given amount of the effect, are evidently Mathematical questions and to be treated, therefore deductively. The Deductive Method and not the Inductive Methods then are applicable in case of **Progressive Effects.**

Take for instance, the heteropathic laws of mind,—as when a complex passion is formed by the coalition of several elementary impulses; or a complex emotion by several simple pleasures and pains, of which it is the result without being the aggregate or in any respect homogeneous with them. We can discover these laws by the slow process of studying the simple feelings themselves and ascertaining synthetically, by experimenting on the various conditions to which they are subject, what they are, by their mutual action upon one another, capable of generating.

§ 33. "Exception proves the Law." Is there any Exception to a law?

The notion that there may be a real exception to a general truth arises from neglecting the proper mode of expressing a law. What is called an exception to a general principle is always a case of some other law interfering with it and disguising or destroying its effect.

Every law of causation is liable to be counteracted and apparently frustrated by coming into contact with other laws, the results of which are more or less opposed to its result. A cause always *tends* to produce its effect, counteracting causes may prevent that effect being manifested in the usual form. A cause is always a *tendency*.

An exception to a law implies the co-existence of that law and a counteracting law. *So an exception proves the law.* To call one of two concurrent principles an exception to the other is contrary to the correct principles of nomenclature. An effect of precisely the same kind and arising from the same cause ought not to be placed in two different categories, merely because, there does or does not exist another cause prepondering over it.

XIII.

THE DEDUCTIVE METHOD.

§ 1. **The Deductive Method and its function :—** The Deductive Method is the method applied in finding out the law of a complex effect, from the laws of the different causes of which it is the joint result. The Deductive Method considers separately the causes which enter into the complex effect and computes or calculates that effect *a priori*—from the balance or product of the effects of the different causes which produce it.

§ 2. **The distinct operations involved in Deduction :—**Given any complex phenomenon, the enquirer considers, what laws already ascertained by Induction seem likely to apply to it (in default of known laws, hypotheses are substituted) ; he then computes the effect that will follow from these laws in circumstances similar to the case before him ; and he verifies his conclusion by comparing it with the actual phenomenon.

Therefore the operations are the following :—

(1) Ascertaining the laws of separate causes by direct Induction.

(2) Ratiocination from the simple laws to the complex case *i. e.*, calculating from the laws of the causes, what effect a given combination of them must produce.

(3) Verification by specific experience.

§ 3. There is difficulty in the application of the Deductive Method; but the method is not rendered inoperative thereby. Verification is useful.

The difficulty is in ascertaining the law of each separate cause which takes share in producing the effect. When in every single instance a multitude, often an unknown multitude of agencies are clashing and combining, what security have we that in our computation *a priori* we have taken all these into reckoning? This question has real weight and would be altogether unanswerable, if there were no test by which, when we employ the Deductive Method, we might judge whether an error of this description has been committed or not. Such a test is Verification, without which all the results it can give, have little other value than that of conjecture.

§ 4. Verification and Errors in Deductive Argument :—If in verifying a deductive argument, the effect as computed from the laws of the causes assigned, does not correspond with the facts observed, there must be an error somewhere. If the fact has been accurately observed, the error must lie in the process of Deduction or Computation, or else in the premises; and we have to find it out.

§ 5. Simple Laws of Nature and Deductive Calculation :—

(i) Unless the simpler Laws of Nature are ascertained or hypothetically framed, Deductive Calculation is not possible.

(ii) Any simple Law of Nature is deemed to have gained in point of certainty, by being found to explain some complex case which had not previously been thought of in connexion with it.

§ 6. Different forms of the Deductive Method:—The Deductive Method presents several forms (i) according to the subject-matter to which it is applied; and (ii) according to the mode of its application.

The following are the forms of the Deductive Method according to the subject-matter to which it is applied:—

(1) **The Abstract Deductive Method** deals with the laws of those sciences which are not concerned with causation, and therefore which are not liable to counteraction,—the law of number and extension, for example. The application of it is found in Euclid's Geometry; so it is also called *the Geometrical Method.*

(2) **The Concrete Deductive Method** deals with those sciences which are concerned with phenomena of causation.

The following are the forms of the Deductive Method according to the mode of its application:—

(1) **The Direct Deductive Method** is that in which we obtain our conclusion or complex law by Deduction first *i. e.*, by a calculation of the effects of the conjoint causes, and afterwards verify it by comparison with the results of experience. This is also called *the Physical Method.*

(2) **The Inverse Deductive Method** is that in which we obtain our law more or less conjecturally by Direct experience and afterwards verify it by showing that it is deducible from more general or better known laws. This is sometimes called the Historical Method, because it is more useful than any other in explaining the movements of history and in verifying the generalisations of Political and Social sciences.

When the forces determining a phenomenon are too numerous or too indefinite to be combined in a direct deduction, we may begin by collecting an empirical law of the phenomenon and then endeavour by deductions from a consideration of the circumstances and forces known to be operative, that such a law was to be expected.

§ 7. The Geometrical Method and Politics :— Politicians while applying the deductive reasoning, sometimes omit to test their results by any comparison with the facts; arguing from certain "Rights of man" or "Interests of classes" or "Laws of Supply and Demand," that this or that event will happen or ought to happen, without troubling themselves to observe whether it does happen or ever has happened. This method of Deduction without any empirical verification is called by Mill the Geometrical Method; and it can be trustworthy only where there is no actual conflict of forces to be considered. In pure mathematical reasoning about space, time and number, provided the premises and the reasoning be correct, verification by a comparison with the facts may be needless, as there is no possibility of counteraction. But when we deal with actual causes, no computation of their effects can be relied upon without comparing our conclusions with the facts: not even in Astronomy and Physics, least of all in Politics."

§ 8. The use of the Historical Method is not confined to the studies of History, Political Science and Sociology.—Though the use of the Historical Method is more useful than any other in these subjects but "we should not allot to each subject its own method and

forbid the use of any other; as if it were not our main object to establish truth by any means. Wherever the forces determining a phenomenon are too numerous or too indefinite to be combined in a deductive demonstration, there the Historical Method is likely to be useful; and this seems often to be the case in Geology and Biology, as well as in the science of History or Sociology and its various subsidiary studies."

§ 9. Direct Deductive Method and Inverse Deductive Method Contrasted:—

In the Direct Method, a deduction is verified by comparing it with an induction or an experiment; in the Inverse Method, deduction is called in to verify a previous Induction.

§ 10. Physical Method and Historical Method Contrasted:—

Ordinarily the Direct Deductive Method is called Physical; and the Inverse Deductive Method is called Historical; but properly speaking the method used in the study of physical phenomena, where causes are comparatively definite is Physical; and that used in the study of Historical phenomena, where causes are indefinite, is Historical. The essential difference between the Physical and Historical Methods is that the Physical Method leads to definite results—amounts to exact demonstration, whereas the results arrived at by the Historical Method are at best only vague and probable.

XIV.

RELATION OF INDUCTION TO DEDUCTION.

§ 1. The Relation between Induction and deduction :—There are three principal stages through which we pass in our Inductive reasoning:—(i) Observation of particular facts and Formation of Hypotheses, (ii) Application of Inductive Methods and Generalization, and (iii) Verification. We know that the Deductive reasoning is involved in Inductive Methods; and Verification is nothing but Deduction. Again the major premise of a deductive reasoning is generally inductively obtained. So there is no opposition between the Deductive and the Inductive reasoning.

Such antithesis is only possible when Induction is regarded as founded on enumeration; and this, we know, can give us no truly universal propositions. But on the view of Inductive Methods, it is clear that the inference is based on deductive principles throughout. It is a derivation of conclusions from hypothetical premises, and nothing can be more deductive than the connexion of a hypothesis with the consequences by which it is verified.

The distinction between Induction and Deduction is one of aspect. "In Induction, reality presents itself in concrete and partially isolated instances, and the task of inference is to discern the universal which is more or less hidden in those instances. In Deduction, on the other hand, reality presents

itself in its universal aspect, and the task of inference is to trace the presence of the universal in the differing and complex instances of its manitestation "—Welton.

§ 2. **Carveth Read on the Subject :**—"Deduction depends on Induction, if general propositions are only known to us through the facts. Induction depends on Deduction; because one fact can never prove another, except so far as what is true of the one is true of the other and of any other of the same kind; and because to exhibit this resemblance of the facts, it must be stated in a general proposition."

§ 3. **Jevons on the importance of Induction and Deduction :**—

"It cannot be said that the Inductive process is of greater importance than the Deductive process, because the latter process is absolutely essential to the existence of the former. Each is the complement and counterpart of the other."

"Induction is the inverse operation to Deduction and cannot be conceived to exist without the corresponding operation, so that the question of relative importance cannot arise."

"It must be allowed that in Logic, Inductive investigations are of a far higher degree of difficulty, variety and complexity than any question of Deduction; and it is this fact no doubt which had led some logicians to erroneous opinions concerning the exclusive importance of Induction."—Principles of Science.

§ 4. **Induction is mainly concerned with observation; Deduction not so.**—In Induction, we are more concerned with the minor premise than with the major one. Here we try by observation and experiment to obtain materials

for a premise which with the Law of Causation or any of the canons of experimental enquiry form a valid syllogism.

Induction is the process of finding the minor premise by observation and experiment, when the major premise is either the law of causation or any of the canons of Inductive enquiry.

Deduction is more concerned with the general propositions. It has not much to do with observation and experiment, but only develop the evident significance of the premises. It brings together the explicit meanings of the given propositions and tries to join together these explicit meanings and thereby to find out what is implicit in their combination.

§ 5. The relation expressed by metaphors:—The expressions of Bacon,—'ascending' and 'descending' respectively for Induction and Deduction, have met with considerable favour. This is a metaphor. There is a whole vocabulary of expressions in common usage which rest upon the same analogy: thus we speak of 'rising to first principles,' 'coming down to particulars,' 'hights of abstraction' and so forth. The notion conveyed by these metaphors is presumably this:—He who is in possession of a generalization is like a man on the top of a hill; not merely on account of the toil expended in getting there, for we may expend trouble as well in digging as in climbing, but on account of the increased powers of vision or insight, which he thus acquires. And the broader the generalization, or the higher we climb, the more pronounced do these advantages generally become.

§ 6. Induction and Deduction; from Effects to Causes and from Causes to Effects:—

It is sometimes said that in Induction we proceed from

effects to causes, whereas in Deduction we proceed from causes to effects. This statement is true, if we accept 'the popular scientific' account of causation. It would not continue to hold good when the cause and the effect are both very loosely defined; and would equally fail when they are defined with the utmost conceivable accuracy. That is, it seems to require that, relation between cause and effect should not be a simply reciprocal one. For instance where the consequences of an illness or a wound are precise and definite and could not be produced by any other injury, our inference is equally certain, and is as naturally couched in the deductive form, whether we start from the cause to infer the effect or conversely. On the other hand when in accordance with popular usage, we omit many of the antecedent elements, our argument in each case alike claims no greater force than that afforded by Analogy or Problablity.

When we start with the assumption that every event is preceded by some one or other of certain groups of phenomena, any one of which would certainly cause that event, we see that the relation between the two elements (the antecedent and the consequent) is no longer reciprocal. Given the antecedent, the consequent necessarily follows, and the inference can easily be thrown into the techneeal deductive form:—All X is followed by Y; this is an X; therefore this will be followed by Y. But given the consequent, we can only conjecture the antecedent.

If we are to determine with certainty which of the possible known cases was productive in the case in question it can only be by an inductive process; that is, we must set to work by

employing Inductive Methods. Although therefore we can not correctly say that arguing from effects to causes is Induction, it is nevertheless true that in the current logical sense of the word "cause," this procedure will almost always demand an appeal to Inductive process, whilst the converse procedure will not do so.

§ 7. Induction and Deduction: From Facts to Ideas and From Ideas to Facts.

Buckle says that in Induction we reason *from facts to ideas*, and in Deduction *from ideas to facts*. In so far as this is another way of estimating that Induction involves generalization, it is true enough; but it sacrifices accuracy and completeness to brevity. Do we never reason *from ideas to ideas* or *from facts to facts?*

There can be no doubt that we sometimes reason from facts to facts. As Mill has remarked, much of the almost unconscious inference of daily life is carried on in this fashion, and inductively. And again the highest kind of reasoning would have to be described as reasoning from ideas to ideas. For example, when we deal with a somewhat abstract subject-matter, we reason in this way.

But what the above expression means, is this: that the law in which we sum up generalization of a number of facts belongs to the class of subjective or mental acquisitions. It involves more of the subjective or mental element than is involved in what are commonly called "facts." We must not forget that "laws" are sometimes called "facts," but that is because they are the compendious statement of all particular facts indicated by them. We may say that Induction involves

generalization from individual observations and Deduction involves specialization to narrow and even to individual results from given generalizations.

§ 8. Induction and Deduction: Analysis and Synthesis.

Induction is called *Analysis*, because from special cases we proceed to find out the fundamental. Induction is the analysis of intension or properties. Deduction is called *synthesis*, because in Deduction we pass from the general law to particular facts,—so from the fundamental property to *all* properties found in particular cases.

This description is true from the intensive point of view; but from the extensive or denotative point of view Induction ought to be regarded as *Synthesis* and Deduction *Analysis*.

XV.

DEMONSTRATION.

§ 1. **Definition of Demonstration** :—A demonstration in the logical sense is a reasoning in which the conclusion follows *necessarily* from the premises,—as opposed to one in which the conclusion is only probable.

§ 2. **The Ground of Demonstration** :—The ground of Demonstration is the Law of Identity. Demonstration is possible only when the conclusion can be shown to be in some way identical with or contained in the premises; in other words when the conclusion is wholly or partially identical with the premises or follows from them as a consequence of something contained in them.

§ 3. **Demonstration and Necessary Truth** :—A Necessary Truth according to the common definition, is such as is supposed to be independent of the evidence of experience. Philosophers belonging to the Experience School *i. e.*, those who hold that the source of all our knowledge is experience, deny the existence of necessary truths in this sense.

But Mill has defined Necessary Truth as that which necessarily follows from assumptions which, by the conditions of the enquiry, are not to be questioned. What is really meant here by *necessity*, therefore, is *certainty of Inference*.

§ 4. **Demonstration and Deductive Reasoning** :—All deductive proofs are regarded as demonstrative

proofs. The propositions of Euclid are said to be demonstrated; and this means that the conclusions are proved by bringing each case under the sweep of the fundamental principles of the science. Demonstration gives us the highest certainty. When a thing is demonstrated, all our doubts vanish at once, because then the thing is seen to follow necessarily from certain fundamental principles or laws whose validity no one questions. Demonstration always presupposes belief in the premises with which we start, whatever may be the rational justification of those beliefs.

§ 5. Deductive Conclusions and Necessary Truths:—Deductive conclusions necessarily follow from the given premises. According to Mill their necessity is only a hypothetical necessity. The Deductive reasoning starts from the granting of certain fundamental suppositions and then proceeds to trace the consequence of such suppositions *i. e.*, what inferences might be drawn from them, leaving for separate considerations how far they are true and what corrections must be made if they are not exactly true. Mill maintains that the major premise of every deductive reasoning is an inductive conclusion;—and if the major premise do not state a necessary truth, the deductive conclusion can not also do so. But the fundamental suppositions are not always inductive conclusions and inductive conclusions are not necessarily uncertain.

§ 6. Inductive Conclusions and Necessary Truths:—We must know first how the inductive conclusions are arrived at, and then only can we say whether those conclusions are the statements of necessary truths or not.

If our induction is only an Induction by Simple Enumeration, our conclusions are only more or less problematic. But according to the view of Scientific Induction, every inductive generalization consists in passing from observed phenomena to their essential and invariable conditions. When this is done, and when it is shown that any new phenomena are determined by just those conditions, all that needs be done is done. There is no inference from some *observed instances* to *all* cases *i. e.*, inference is not based there on enumeration ; for once it is established that the essential conditions in the several cases are identical, no place remains for such an inference ; we have a simple example in the Law of Identity. When the conditions of a phenomenon are accurately stated,then the general judgment is necessarily true. If our analysis is *inaccurate*, if what we assume to be the conditions of the phenomenon, are not really its conditions, then our general propositions are not true. If our analysis is *inadequate*, if what we take for the essential conditions, include other elements or if we omit some essential elements of the conditions, then our judgments has not universal validity. In so far as any judgment is *true*, it is *necessarily true*, for necessity consists only in the connection of a consequent with its ground. When *accuracy* and *adequacy* are undoubted, there is no *uncertainty*.

§ 7. **Why Mathematical Truths are particularly necessary** :—Mathematical truths are often held to be of a superior rank to those of Physical Science ; the former are spoken of as "necessary," the latter as 'contingent truths.' The character of necessity *i. e.*, the peculiar certainty attributed to Mathematical truths, is according to

Mill, an illusion. Mill is wrong in holding that the certainty of Mathematical truths is fictitious.

Mathematical Truths are not necessary in the psychological sense that everybody must think them as truths; they are necessary in the sense,—those who think them at all, can think them only in one way. "In Mathematics, we deal entirely with abstract notions; we, therefore, constitute the conditions by our own mental act and as so constituted they are not liable to be modified by other conditions. In Geometry, for example, we deal only with abstract ideas of limits,—with surfaces, lines and points. We construct our figures by a mental synthesis of just those elements and nothing more, and from such construction, we derive propositions which, as they express relations subject to no conditions but those which we have ourselves imposed, are applicable to every other figure in which just those conditions and those only are fulfilled, and are, therefore, universal." The necessary character of Mathematical truths is therefore due, to exact knowledge of conditions, but such complete knowledge is not always possible in the physical science. Still all general propositions about nature are necessary, in so far as they are true.

§ 8. Are the Concepts and Axioms, on which Mathematics rests, generalizations from sensuous experience?

According to Mill, they are so; but points and lines are not sensations or combination of sensations,—they are the conceived limits of surface, in other words relations which exist for thought but not for sense-perception. Mill says in one place

that there exist no points without magnitude; no lines without breadth, or perfectly straight; no circles with all their radii exactly equal; nor squares with all their angles perfectly right. How can he again say that the points, lines, circles and squares which any one has in his mind, are simply copies of the points, lines, circles and squares which he has known in his experience?

The concepts and axioms, on which Mathematics rests, are not generalizations from sensuous experience. Mill's theory of knowledge has led him to think them to be otherwise. The source of knowledge, according to Mill, is sensuous experience. He does not properly acknowledge the function of our intellect.

§ 9. Whewell and Mill's controversy regarding the Mathematical Truths.

According to Whewell, the truth of the Mathematical axioms is perceived *a priori;* repeated trials are not required to verify their truth; they may be suggested by experience, yet experience does not prove them.

Whewell's argument (a):—If an axiom were proved by the evidence of the senses, we could only be convinced of its truth by actual trial, as is the case of "two straight lines cannot enclose a space," by seeing or feeling the straight lines; but it is seen to be true by merely thinking of them; therefore the ground of belief must be in the laws of the mind itself.

Mill's reply:—Imagination can so perfectly reproduce sensations of *form*, that our mental pictures of lines, circles &c are just as fit subjects of experiment as the external pictures or the realities themselves.

Whewell's argument (b) :—The evidence of Axioms from actual ocular inspection is not only unnecessary, but unattainable. The thing asserted being that the two parallel lines will not meet in infinity,—how can the senses take cognisance of a non-existent phenomenon ? Can we see or feel the lines *not meet* at an infinite destance ?

Mill's answer :—We know that if two parallel straight lines ever meet or even begin to approach, this must take place at some finite distance.

Whewell's argument (c) :—Axioms are conceived by us not only as true, but as universally and neccessarily true. Now experience cannot possibly give to any proposition this character. I have seen snow a hundred times and may have seen that it is white, but this cannot give me entire assurance that all snow is white,—much less that snow must be white. Experience cannot offer the smallest ground for the necessity of a proposition. Necessary truths are those in which we not only learn that the proposition *is true*, but see that it *must be true* ; in which the negation of the truth is not only false but impossible, in which we cannot even by an effort of imagination conceive the reverse of that which is asserted.

Mill's answer :—Inconceivability of the contradictory of a proposition is not a mark of necessary truth, nor even a certain mark of its being true at all.

Inconceivability is an accidental thing dependent on the mental constitution and history of the person who tries to frame the conception. Our capacity or incapacity of conceiving depends on our association. Educated minds can break up their associations more easily than the uneducated.

We have several examples of propositions, once regarded as inconceivable by the greatest men, now regarded not only as conceivable, but as the only true accounts.

Sir William Hamilton agrees with Mill in rejecting inconceivability as a certain mark of falsity. He says that what is inconceivable may be true. He gives an example. "Matter must be either infinitely divisible or not," by the Law of Excluded Middle. Both of them are inconceivable; but one must be true. ∴ What is inconceivable may be true.

There are other arguments in favour of the *a priori* view, but Mill has tried to refute them also.

Argument (i):—Increase of certainty *pari passu* with increased experience is a mark of a truth derived from experience.

After seeing 10 people die, I should expect more confidently the mortality of any new man than if I had seen five only; still more if I had seen one hundred than if merely ten and so on. Axioms want this mark, being believed with the fullest certainty, immediately they are understood.

Mill's reply:—After having once arrived at full certainty, no further experience can increase that certainty. In axioms and similar assertions a single experience is sufficient.

Argument (ii):—Impossibility of establishing a proposition by propositions simpler or more certain than itself is a mark of the necessary truth of that proposition. Axioms possess this mark and therefore are necessarily true.

Mill's reply:—The propositions which are admitted to be proved by experience cannot also be established by propositions simpler or more certain than themselves.

Argument (iii) :—Mankind universally and constantly acting as if they believed them, is a mark of necessary truth. Axioms possess the mark.

Mill's reply :—Such belief is found in propositions proved by experience also.

Argument (iv) :—There is to our mind a distinct and conscious difference between the two classes of truths both as to their certainty and the kind of evidence we should bring forward, if they are disputed.

(*a*) Two and three make five, or two straight lines can not enclose a space. (*b*) Fire burns or a stone dropped into water goes to the bottom of it. The former seems to have a necessity about it which does not belong to the latter.

Mill's reply :—This argument asserts the inconceivableness of the contradictory in different words.

XVI.

SYLLOGISMS. FUNCTIONS OF SYLLOGISMS.

§ 1. **Definition of Syllogism** :—A syllogism is a process of reasoning in which we pass from two given propositions containing a common term to a new proposition, whose truth follows from theirs as a necessary consequence.

The original meaning of "Syllogism" was "computation." Aristotle borrowed it from Mathematics.

§ 2. **Premises and Conclusion** :—The two propositions from which we start in a syllogism are called *premises;* one of them is called the major premese, and the other the minor premise. The new proposition which is obtained from them is the *conclusion.*

§ 3. **Different kinds of Syllogisms** :—Syllogisms are *pure*, when both the premises are of the same character, *i. e.*, (i) when both the premises are categorical, (ii) When both are hypothetical, or (iii) when both are disjunctive. Syllogisms are mixed when both the premises are of different character,—(i) When the major premise is hypothetical and the minor categorical, (ii) when the major premise is disjunctive and the minor categorical, (iii) when the major

premise is hypothetical and the minor disjunctive. So we have :—

Syllogisms :—

- 1. Pure :—
  - 1. Categorical.
  - 2. Hypothetical.
  - 3. Disjunctive.
- 2. Mixed.—
  - 1. Hypothetical.
  - 2. Disjunctive.
  - 3. Dilemmas.

§ 4. Rules of Syllogism :—

I. Rules giving the description of Syllogism, or relating to its Nature :—

(i) A syllogism must contain three and only three terms.

(ii) A syllogism must consist of three and only three propositions.

II. Rules of Distribution or relating to Quantity :—

(i) The middle term must be distributed once at least.

(ii) No term may be distributed in the conclusion which was not distributed in one of the premises.

III. Rules of Quality :—

(i) From two negative premises no conclusion follows, *i.e.*, one of the premises must be affirmative.

(ii) If one premise is negative, the conclusion must be negative; and to prove a negative conclusion, one of the premises must be negative.

§ 5. Figures of Syllogism :—Syllogisms may be divided into four figures according to the position of the middle term in the premises. Figure is the form of a syllogism as determined by the position of the middle term in the two premises. In the first figure, the middle term is the subject in the major premise and predicate in the minor. In the

second figure it is the predicate in both the premises. In the third figure, the middle term is the subject in both of them. In the fourth figure it is the predicate in the major premise and subject in the minor.

§ 6. The Basis of Syllogistic Reasoning : Axioms of Four Figures :—

All Syllogistic reasoning is based upon the fundamental Laws of Thought.

Axiom of the First Figure :—Whatever is predicated whether affirmatively or negatively, of a term distributed may be predicated in like manner of everything contained in it. This is the *Dictum de omni et nullo.*

Axiom of the Second Figure :—If one term is contained in and another excluded from a third term, they are mutually excluded. This is the *Dictum de diverso.*

Axiom of the Third Figure :—If anything, which is stated to belong to a certain class, is affirmed to possess or to be devoid of certain attributes may be predicated in like manner of some members of that class. This is the *Dictum de Exemplo.*

Axiom of the Fourth Figure :—Three classes cannot be so related ; that the first is wholly included in the second ; the second wholly excluded from the third ; and the third partly or wholly included in the first. This is the *Dictum de reciproco.*

§ 7. Each figure has its peculiar importance.—

According to Lambert, the first figure is suited to the discovery or proof of the properties of a thing ; the second is the discovery or proof of the distinctions between things ; the

third to the discovery or proof of instances and exceptions the fourth to the discovery or exclusion of the different species of a genus.

First Figure :—In the first figure (i) the deductive character of syllogistic reasoning,—the subsumption of a special case under a general rule is expressed in the very form of the argument; so deductions from general scientific principles are expressed in it. (ii) We have all kinds of conclusions *i. e.*, A, E, I and O propositions. (iii) A universal affirmative proposition can be obtained only in this figure. The business of deductive sciences is mainly to establish universal affirmative propositions; so the first figure is very useful for this purpose.

Second Figure :—In this figure negative conclusions only can be proved; so it is most employed in arguments intended to disprove some assertion.

Third Figure :—In this figure particular conclusions only can be proved; it is therefore, specially adaped to the establishment of exceptions to a general rule.

Fourth Figure :—The chief value of the fourth figure is theoretical; as it is a possible arrangement of terms, its recognition as such is necessary to complete the formal, doctrine of figure.

The relation of species and genus would be much more satisfactorily established by a syllogism in the first figure in which the name of the species is the minor and that of the genus the major term, than by one in the fourth figure in which the major term represents the species and the minor term the genus.

§ 8. The Axiom involved in Syllogistic Reasoning: Dictum de omni et nullo :—

According to the Scholastic logicians the first figure is the perfect type of Categorical syllogism. All other forms of syllogism, if they can be reduced to this figure, are regarded as valid. Therefore the axiom involved in this figure i.e., *Dictum de omni et nullo* is considered as the fundamental axiom or principle of syllogism.

§ 9. Mill's Remarks on the "Dictum de omni et nullo" :—

This dictum is a mere identical proposition and not the fundamental axiom of syllogism. A whole class is in fact the same thing as all the individuals included in it. Therefore if we say "whatever is true of *a whole class* is true of *every individual in that class*," it is the same as saying 'whatever is true of *all the individuals of a class* is true of *every individual in it*,"—an obviously identical proposition.

§ 10. Mill's Axiom of Syllogism :—According to Mill, every Syllogism comes within the following formula :—

Attribute A is a mark of the attribute B.
The given object has the mark A.
∴ The given object has the attribute B.

The Syllogism :—All men are mortal.
Socrates is a man.
∴ Socrates is mortal.

will be expressed in the following manner :—

The attribute of men is a mark of the attribute mortality.
Socrates has the attribute of man.
∴ Socrates has the attribute mortality.

Therefore the axiom involved in syllogistic reasoning is this :—" Whatever has any mark, has that which it is a mark of."—Or, when the minor premise as well as the major is universal, we may state it thus : " Whatever is a mark of any mark, is a mark of that which this last is a mark of." So the axiom is,—" Nota notæ est, nota rei ipsius."

Remarks :—This axiom is given by Kant. Mill has rejected the old dictum and accepted this one. Mill need not reject the "dictum de omni et nullo", thinking it to be an identical proposition. If the dictum be an identical proposition, then the axiom of " Nota Notæ " is also an identical proposition. In fact the old axiom is stated from the *denotative* point of view, and the new one is stated from the *connotative* point of view.

§ 11. Mill's objections against the Syllogistic Reasoning :—

(i) Syllogism is not the usual process of reasoning.

(ii) The fallacy of Petitio Principii is involved in every Syllogism.

§ 12. Syllogism and the usual process of reasoning :—

Archbishop Whately has contended that Syllogism is *the* mode in which all men reason and must do so, if they reason at all. In syllogism we reason from universal to particular According to Mill, the usual process of reasoning is from particular to particular. Mill says :—" If from our experience of John, Thomas &c., who once were living, but are now dead, we are entitled to conclude that all human beings are mortal. we might surely without any logical consequence have conclu-

ded at once from these instances that the Duke of Wellington is mortal. The mortality of John, Thomas and others is after all, the whole evidence we have for the mortality of the Duke of Wellington." Mill also adds: —"All inference is from particulars to particulars; General propositions are merely registers of such inferences already made, and short formulæ for making more. The major premise of a syllogism consequently is a formula of this description and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according to* the formula: the real logical antecedent or premise being the particular facts from which the general proposition was collected by Induction."

Criticism of Mill's View:—" It is true that the Syllogism is not the process by which we usually reason. But it is equally true that our usual reasonings will not be valid and therefore not deserve the name, unless they are capable of being reduced to the syllogistic form. Mill seems to make a confusion between the business of Psychology and that of Logic. It is not the business of the latter to give an account of the various processes by which people *reason* correctly or incorrectly, but to give an account of the processes by which they *ought to reason* and must reason if they wish to reason correctly"—Dr. Ray.

§ 13. What is meant by the fallacy of Petitio Principii?

When the truth of any of the premises, from which the conclusion is drawn, is known or derived from the truth of that conclusion, the fallacy committed in the reasoning is called Petitio Principii.

Or, in other words :—Petitio Principii is a fallacy which we commit when we attempt to prove a conclusion from the premises, the ground of any one of which is the truth of the conclusion.

We must remember that Petitio Principii is a fallacy of proof. We cannot commit it, unless we start from a proposition which we want to prove.

Example :—We come to the conclusion, "All men are mortal," after observing the death of A, B, and C. Then we reason syllogistically :—

All men are mortal.
X is a man.
∴ X is mortal.

In this case there is no Petitio Principii. But suppose we come to the conclusion, "All men are mortal" after observing the death of A, B and X, and then reason syllogistically :—

All men are mortal.
X is a man.
∴ X is mortal.

In this case there is undoubtedly Petitio Principii.

§ 14. Mill's attempt to prove that every syllogism involves the fallacy of Petitio Principii.

According to Mill, the general principle, instead of being given as evidence of the particular case, cannot itself be taken for true without exception, until every shadow of doubt which could affect any case comprised *within* it, is dispelled by evidence. The truth of the conclusion ought to be known before the assertion of the universal premise in the syllogism; no one is warranted in asserting the general proposition with-

out having satisfied himself of the truth of everything which it includes. So the syllogistic conclusion does not give us any new information not given by the premises; and it is not an inference at all. It is drawn from a premise, whose truth depends upon the truth of the conclusion. Therefore there is Petitio Principii in every syllogism.

Criticism of Mill's View:—(i) Mill himself has said that in Induction we come to a general conclusion after the observation of some cases only. How can he say then, when we have the syllogism :—

All men are mortal.
The Duke of Wellington is a man.
∴ The Duke of Wellington is mortal.

that we ought to know that "the Duke of Wellington is mortal," before asserting the proposition, "All men are mortal"? We can establish the proposition, "All men are mortal," after knowing only that "Socrates is mortal," "Plato is mortal," and "Aristotle is mortal," and without noticing the mortality of the Duke of Wellington. We acknowledge, that the premises of a valid syllogism cannot be true unless the conclusion is also true; but we nevertheless have valid grounds of accepting the premises, which are independent of any explicit knowledge of the truth of the conclusion. So there is no *petitio principii* in the syllogism in which we have the conclusion :—The Duke of Wellington is mortal. The major premise is not a mere summation of instances, as Mill has said.

(ii) Again a syllogistic inference requires the combination of both premises, but Mill's objection involves the tacit

assumption that the minor premise is unnecessary. It is when the relations predicated in the two premises are brought before the mind, then it sees the force of the inference. Mill says,—"When you admitted the major premise, you asserted the conclusion." If so, the minor premise is superfluous. But Mill himself acknowledges that the major premise does not individually specify all it merely includes but only indicates them by marks and that the function of the minor premise is to compare any new individual with the marks. The necessity to a syllogistic inference, of the minor premise, is then a proof that such an inference is not a *petitio principii.*

(iii) It is also to be mentioned that it is possible to accept the premises without drawing the conclusion ;—the conclusion of a syllogistic reasoning makes explicit in thought what is implicit in the premises. This is evident in Geometrical conclusions. So syllogistic conclusions can increase our knowledge. The function of inference is not only to convey some new knowledge to us which are not contained in the premises, but also to make explicit what is implicit in them.

§ 15. The Use and Function of Syllogism :—

I. We know that the syllogistic reasoning and the Inductive reasoning are equally important and there are inferences in both of them. An Inductive conclusion conveys new information ; a syllogistic conclusion also increases our knowledge by making explicit in thought what is implicit in the premises.

II. (*a*) But according to Mill, both the major premise and the conclusion of every syllogism are inductive inferences based upon particular facts. The major premise is a general

proposition. The advantage in judging whether any controverted inference is legitimate, of referring to a parallel case, is universally acknowledged. But a general proposition is nothing but all possible parallel cases at once, all cases to which the same set of evidentiary considerations is applicable. The use of the syllogism is in truth no other than the use of general proposition in reasoning. A syllogism is not a form in which we *must* reason, but it is a form in which we *may* reason and into which it is indispensable to throw our reasoning, when there is any doubt of its validity. So syllogism is a mode of verifying any given argument.

Mill says :—"The conclusion in a syllogism is not drawn *from* the major premise, but *in accordance with* it." Both the major premise and the conclusion are the inferences drawn from the observed particular facts. The major premise is not the ground of the conclusion ; but the particular facts observed are the grounds of both of them. In a syllogism, we examine the *consistency* of the two inductive conclusions. The major premise is a general proposition and consequently the embodiment of several inductive conclusions. So when any syllogistic conclusion is shown to be consistent with the major premise ; the conclusion is also confirmed by it.

(*b*) The syllogistic operation, according to Mill, is not a process of inference, but a process of interpretation. In a syllogism we have to avoid an inconsistency between the major premise and the conclusion. With this view, we interpret our general proposition. The syllogism is thus merely a process by which the real or complete meaning of a general proposition established by Induction is made explicit, and by

which the validity of a reasoning is tested. It is in other words an interpreter of the general proposition and a test of reasoning. The rules for this interpretation are the rules of the syllogism they merely help us in interpreting correctly the true meaning of general propositions and in applying them to particular cases.